Hybrid aeroacoustics using Helmholtz decomposition

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**Motivation**

**Flow and Acoustics**
- Solve equations of fluid-dynamics
- Flow + Acoustic field (e.g. $p = \tilde{p} + p'$)

**Acoustic analogy**

Wave propagation model (*reformulation of flow equations*):

$$\Box p' = \text{RHS}(p, u, \rho, \ldots) = \text{RHS}(\tilde{p}, \tilde{u}, \tilde{\rho}, p', u', \rho', \ldots)$$

Non-radiating base flow (*e.g. incompressible Flow, (Goldstein, 2003)*):

$$\tilde{\Box} p' = \text{RHS}(\tilde{p}, \tilde{u}, \tilde{\rho}, \ldots).$$

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Aim of the presentation

Show how to avoid the ambiguity of the hybrid aeroacoustic workflow (separates flow and acoustic simulation) by a non-radiating base flow (vortical projection).

1. Compressible flow simulation \((U_\infty = 50 \text{ m/s}, \ M = 0.14, \ Re = 29000)\)^a
2. Helmholtz decomposition \(\rightarrow\) non-radiating base flow
3. Acoustic propagation

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Show how to avoid the ambiguity of the hybrid aeroacoustic workflow (separates flow and acoustic simulation) by a non-radiating base flow (vortical projection).

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Measurement: Pressure C1
All spatial dimensions in mm

$U_\infty = 50 \text{ m/s}$
$\delta = 10$
$8.7$
$3.3$
$15.9$
$24.7$
$15.9$
$8.7$

$L_w = 8.76 \text{mm}$
$L = 15.9 \text{mm}$
$D_w = 3.3 \text{mm}$
$D = 24.7 \text{mm}$
$W = 15.9 \text{mm}$
Compressible, air, DES flow simulation

CFD grid 12 Mio cells

Vorticity
Compressible, air, DES flow simulation *(grid convergence study)*

1\textsuperscript{st} shear layer mode $f_{R_1}$

Helmholtz res. $f_H$

Cavity res. $f_{C_y}$

Analytical calculation: $f_{R_1} = 1733\, \text{Hz}$, $f_H = 2149\, \text{Hz}$, $f_{C_y} = 3102\, \text{Hz}$

Frequency (Hz)

Pressure level (dB) with $p_{\text{ref}} = 20\, \text{Pa}$

- G-C (2.9 Mio)
- G-M (5.7 Mio)
- G-F (11.3 Mio)
- Henderson 50m/s

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Objectives

Hybrid aeroacoustic workflow:

- ✔ Compressible flow simulation
- 2 Helmholtz decomposition → non-radiating base flow
- 3 Acoustic propagation
Helmholtz decomposition

Decomposition of the velocity field into vortical part $A_v \in H(\text{curl}, \Omega)$, compressible part $\phi_c \in H^1(\Omega)$, and potential flow part $u_\perp$ in the space of harmonic functions$^1$.

**Helmholtz decomposition**

\[
\mathbf{u} = \mathbf{u}_v + \mathbf{u}_c + \mathbf{u}_\perp = \nabla \times A_v + \nabla \phi_c + u_\perp
\]

**Vortical part – Neumann problem, Mass regularization, Nedelec-Finite Elements**

\[
\nabla \times \nabla \times A^*_v = \nabla \times \mathbf{u} = \omega
\]

\[
A^*_v \in \mathcal{W} = \{ \varphi \in H(\text{curl}, \Omega) | \mathbf{n} \times \nabla \times \varphi = \mathbf{n} \times \mathbf{u} \text{ on } \Gamma \}\]

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Helmholtz decomposition

Vortical velocity field
- $u^*_v = \nabla \times A^*_v$
- Shear layer
- Turbulence

Advantage Vortical part (Schoder, 2017)
- Simple boundary conditions
- No singularities at reentrant corners

Helmholtz decomposition

Compressible velocity field

\[ \mathbf{u} = \nabla \times \mathbf{A}_v + \nabla \phi_c + \mathbf{u}_\perp \quad |\nabla \cdot \mathbf{u}| \]

\[ \Delta \phi_c^* = \nabla \cdot \mathbf{u} \]

- \( \mathbf{u}_c^* = \nabla \phi_c^* \)
- Singularities at reentrant corners

Advantage Compressible part

Fast computation during the flow simulation (pressure correction equation)
Objectives

Hybrid aeroacoustic workflow:

- Compressible flow simulation
- Helmholtz decomposition → non-radiating base flow
- Acoustic propagation
Vortex sound \( (\text{Howe, 1975}) \)

\[
\frac{1}{c^2} \frac{d^2 v}{dt^2} H - \nabla \cdot \nabla H = \nabla \cdot (\omega \times u^v) \quad \text{with} \quad \frac{dv}{dt} := \frac{\partial}{\partial t} + u^v \cdot \nabla
\]

- Linearized around the vortical flow

Conclusion

Objectives

Hybrid aeroacoustic workflow *separates flow and acoustic simulation*:

- Compressible flow simulation $\rightarrow$ *aeroacoustic feedback*
- Helmholtz decomposition $\rightarrow$ *non-radiating base flow*
- Acoustic propagation

Outlook

Hybrid aeroacoustic workflow *separates flow and acoustic simulation*:

1. Study of the neglected interaction terms
2. Helmholtz decomposition, increase performance of AMG
3. First investigations at $M=0.8$

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Content

Hybrid aeroacoustic workflow (separates flow and acoustic simulation):

1. Compressible flow simulation $\rightarrow$ *aeroacoustic feedback*
2. Helmholtz decomposition $\rightarrow$ *non-radiating base flow*
3. Acoustic propagation