Graphs in Quantum Mechanics

http://tph.tuwien.ac.at/~svozil/publ/2019-Svozil-Shiraz-pres.pdf

Karl Svozil

ITP/Vienna University of Technology, Austria
svozil@tuwien.ac.at

Shiraz, Iran, May 4th, 2019

# Possible quantum advantages: features not present in classical "paper machines"

▶ Quantum parallelism – aka *coherent superposition* – of classically mutually exclusive bit states (Schrödinger DOI: 10.1007/BF01491891 (§5, cat paradox), 10.1007/BF01491914, 10.1007/BF01491987);

▶ Quantum collectivism – aka (possibly nonlocal correlations DOI: 10.1103/PhysRev.47.777) entanglement – in a multi-particle situation: information encoded only in *relational properties* among particles; individual particles have no definite property; exploitable for quantum cryptography & communication & authentification (Schrödinger DOI: 10.1007/BF01491891, 10.1007/BF01491914 (§10), 10.1007/BF01491987);

Possible quantum advantages: features not present in classical "paper machines" cntd.

- ▶ Quantum probabilities based on vectors (orthogonal projection operators) rather than on sets: non-classical expectation values rendering different (from classical value assignments) predictions; in particular, violations of Boole-Bell type inequalities; exploitable for quantum cryptography & communication & authentification (Boole DOI: 10.1098/rstl.1862.0015, Bell DOI: 10.1103/RevModPhys.38.447);

- ▶ Quantum complementarity: in general quantized systems forbid measurements of certain pairs of observables with arbitrary precision: "you cannot eat a piece of the quantum cake & have another one too;" exploitable for quantum cryptography & communication (Pauli DOI: 10.1007/978-3-642-61287-9, Moore DOI: 10.1515/9781400882618-006);

Possible quantum advantages: features not present in classical "paper machines" cntd.

▶ Quantum value indefiniteness: no classical (true/false) value assignments on certain collections of (intertwining) quantum observables; exploitable for quantum oracles of randomness (Gleason DOI: 10.1512/iumj.1957.6.56050, Kochen & Specker DOI: 10.1512/iumj.1957.6.56050, Abbott, Calude, Svozil DOI: 10.1017/S0960129512000692, 10.1063/1.4931658).
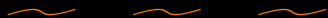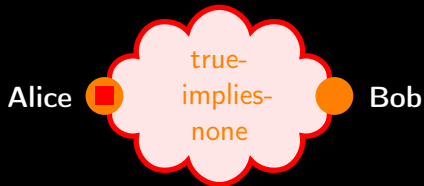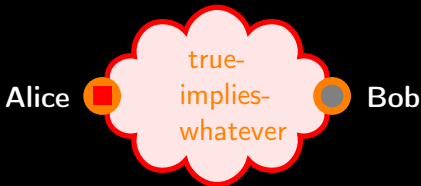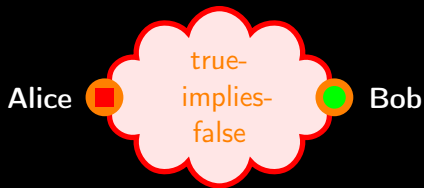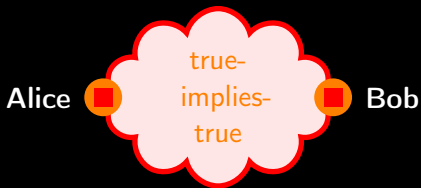
"Babylonian" example collection: Stephen Jordan's
quantum algorithm zoo @ url `http://quantumalgorithmzoo.org/`

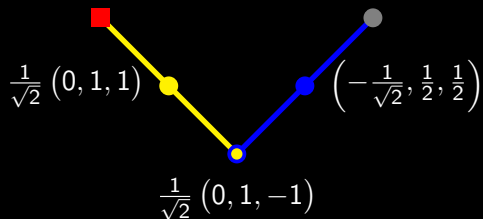# Scheme to exploit quantum value indefiniteness supporting/rendering quantum (oracles for) random number generators

▶ Alice prepares a pure state, representable by a vector (in a context);

▶ Bob measures an observable proposition, representable by another vector (in a context) which is neither collinear nor orthogonal to Alice's preparation.

▶ Alice's and Bob's preparation & measurement are then connected by a quantum cloud – that is, by a collection of intertwining counterfactual quantum contexts (and observables).

▶ These clouds are then interpreted classically; in particular, and in its strongest form, it is shown that these clouds – or at least the outcome of Bob's measurement – do not have any classical representation.
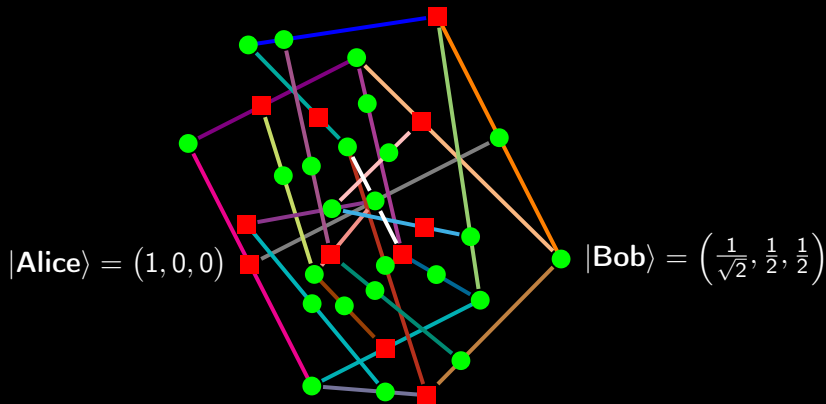
How is |**Bob**⟩ given |**Alice**⟩? True? False? Whatever? None?

$|\mathbf{Alice}\rangle = (1, 0, 0)$ $|\mathbf{Bob}\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$

$\frac{1}{\sqrt{2}}(0, 1, 1)$ $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$

$\frac{1}{\sqrt{2}}(0, 1, -1)$

True (1) implies false (0) (Svozil DOI: 10.3390/e20060406, based on Abbott, Calude & Svozil DOI: 10.1017/S0960129512000692)
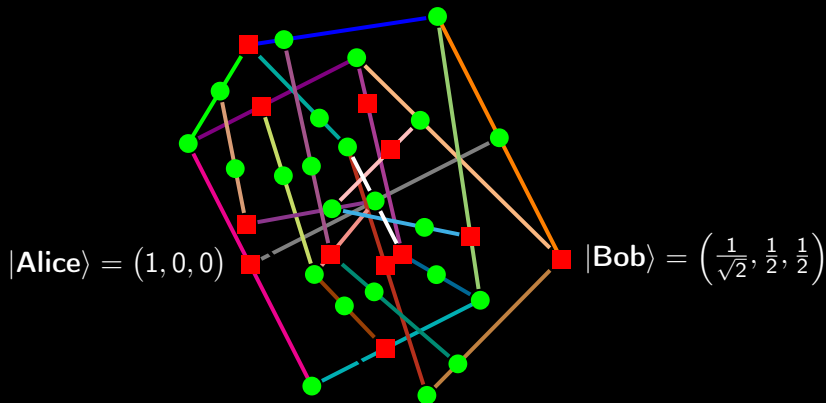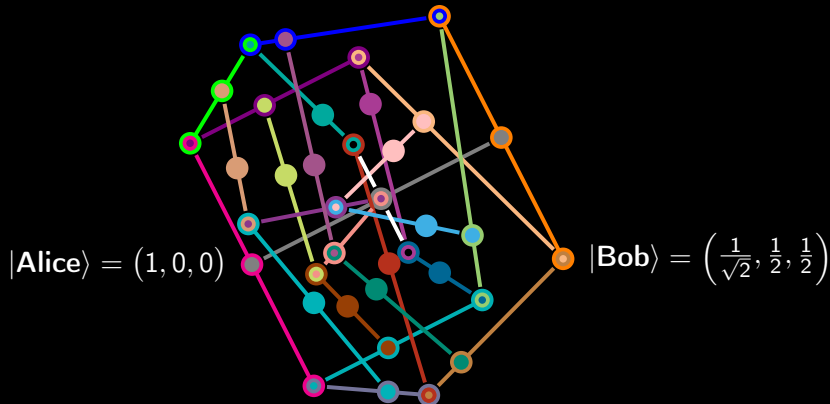

$|\textbf{Alice}\rangle = (1, 0, 0)$

$|\textbf{Bob}\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$

True (1) implies true (1) (Svozil DOI: 10.3390/e20060406, based on Abbott, Calude & Svozil DOI: 10.1017/S0960129512000692)

$|\textbf{Alice}\rangle = (1, 0, 0)$

$|\textbf{Bob}\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2} \right)$

$|\textbf{Alice}\rangle = (1, 0, 0)$

$|\textbf{Bob}\rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$

Strategies to obtain value indefiniteness/partiality

The scheme of the construction & proof of partiality of value assignments is as follows:

(i) Find a logic (collection of intertwined contexts of observables) exhibiting a true-implies-false property on the two atoms **a** and **b**.

(ii) Find another logic exhibiting a true-implies-true property on the same two atoms **a** and **b**.

(iii) Then join (paste) these logics into a larger logic, which, given **a**, neither allows **b** to be true nor to be false. Consequently **b** must be value indefinite.

Extensions of value indefiniteness/partiality

Partiality/value indefiniteness can be extended to any vector **b** non-collinear and non-orthogonal to **a** (Abbott, Calude & Svozil DOI: 10.1017/S0960129512000692)

For a (in some respects weaker because it is based on stronger assumptions) proof relative to global truth assignments, see Pitowsky DOI: 10.1063/1.532334

History of contextual sets & elational properties realizable by two-point quantum clouds

| if a is true classical value assignments	anectodal, historic quantum realisation	reference to utility or relational properties
imply b is independent (arbitrary)	firefly logic L_{12} eg, Cohen, 1989[pp. 21, 22]	
imply b false (TIFS)	Specker bug logic S, 1965 [Fig. 1, p. 182]	Stairs, 1983 [p. 588-589], Cabello et al, 1995 ... 2018
imply b true (TITS)	extended Specker bug logic	KS, 1967 [Γ_1, p. 68], Clifton, 1993 [Sects. II,III, Fig. 1], Belinfante, 73 [Fig. C.l. p. 67], Pitowsky, 1982 [p. 394], Hardy, 1992, 1993, 1997, Cabello et al, 1995 ... 2018
iff b true (nonseparability)	combo of intertwined Specker bugs	KS, 1967 [Γ_3, p. 70]
imply value indefiniteness of b	depending on types of value assignments	Pitowsky, 1998, Abbott et al, 2012 ... 2015

Do clouds "exist" merely in our minds? Do they represent our own subjective imaginations & constructions?

Summary

- ▶ Quantum parallelism exploitable sometimes (similar to zero knowledge proofs) but not always; that is, for all equivalence classes (partitions).
- ▶ Quantum random number generators (oracles) are "theoretically certified" relative to the assumptions made, and the quantum means employed.
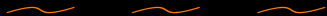
For some critical thoughts on the prospects of quantum computation, please see quantum hocus-pocus DOI: 10.3354/esep00171.

Thank you for your attention!