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An analytical approach for a performance calculation of shuttle-based storage and retrieval systems

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ABSTRACT

This paper deals with a method for performance calculation of shuttle-based storage and retrieval systems (SBS/RS) with tier captive shuttles. A continuous time open queueing system with limited capacity approach is applied. The cycle times of shuttles and lifts can be used directly with their time distributions. These cycle times are determined by a discrete spatial value approach. The main novelty of this approach is the use of a time continuous queueing model to take the interactions between lifts and shuttles combined with a spatial discrete approach for the processes of lifts and shuttles into account. A comparison between the analytical approach and a discrete events simulation is presented to validate the developed model. The data used in this comparison to validate this approach was given by an European material handling provider. Finally, it will be outlined how the model can be used for designing SBS/RS, to meet given requirements.

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KEYWORDS

AS/RS; SBS/RS; queueing system; discrete spatial value; tier-captive autonomous vehicle

1. Introduction

Due to the development of the economy to a demand-oriented delivery and storage of consumables, the requirements for storage technology become higher and higher. In order to meet these requirements, autonomous vehicle storage and retrieval systems (AVS/RS) became a widely used technology. In those systems, there are normally two types of transporters working. The vertical movement is done by lifts. The horizontal movement is done by vehicles called shuttles. There are two different types of AVS/RS. One is called tier-to-tier storage and retrieval system. This type uses shuttles that are able to change tier and to leave the storage system using the lift. Here the lift only transports the shuttles, with or without any tote loaded. In this system, all shuttles can access any location within the system. If more throughput is needed, additional vehicles can be added without changing the rack configuration (Malmberg, 2002). The second type are tier-captive systems. In these systems, shuttles cannot leave their tier. The lift in this type of a shuttle-system transports only a tote and is not able to transport the shuttle itself. This leads to an independency between shuttle and lift movement. This kind of system reaches high levels of performance. This performance can be quantified

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by the reachable throughput of such a system. That's the reason why tier-captive systems are offered by many providers of material handling systems (Marchet, Melacini, Perotti, & Tappia, 2012). These systems are also known as shuttle-based storage and retrieval systems (SBS/RS) (Lerher, Ekren, Dukic, & Rosi, 2015).

The aim of this study is to depict an analytical approach for SBS/RS that is easy to handle and that calculates the maximum throughput of a SBS/RS. Furthermore, this analytical approach should be used for optimizations of the SBS/RS for given requirements. To describe the existing system as real, the considered approach is continuous in time and discrete in space. This paper is organized as follows. The first part in this paper is the presentation of the literature, which is published to the topic of tier-captive shuttle-systems, in Section 2. The treated shuttle-system and the underlying assumptions will be presented in Section 3. In Section 4, the calculation of the performance will be shown. Especially, the interarrival time, the service time of the shuttle and the open queueing model with limited capacity. This approach takes the interactions between lifts and shuttles into account. In Section 5, a numerical study is presented that demonstrates the accuracy of the developed calculation model by comparison to a DES. The parameters for this numerical study are given by an European material handling provider. Finally, in Section 6 there will be a summary of the paper and an outlook on future research.

2. Literature review

Due to the large numbers of design options for SBS/RSs, it is important to know which configuration of SBS/RS is to be implemented.

One way to get accurate performance measures for given system configurations is a discrete-event simulation (DES). Various publications by, e.g., B. Y. Ekren and Heragu (2012), Marchet, Melacini, Perotti, and Tappia (2013), Trummer and Jodin (2014), Kumar, Roy, and Tiwari (2014), Lerher, Ekren, Sari, and Rosi (2015), B. Ekren, Sari, and Lerher (2015) deal with this. The following section will focus on publications that present an analytical approach for single aisle tier-captive SBS/RS to describe the system and to evaluate the throughput: The publication by Heragu, Cai, Krishnamurthy, and Malmborg (2011) analyses a system with random storage policy by modelling it as an open queueing network (OQN). Roy and Krishnamurthy (2011) modelled a semi-open queueing network (SOQN) to consider congestion effects and conveyor loops instead of lifts. Furthermore, methods to calculate average performance measures of tier-captive single-aisle SBS/RS by modelling an OQN are developed by, e.g., Marchet et al. (2012) and Wang, Mou, and Wu (2016). Lerher developed in his two publications (Lerher, 2016; Lerher et al., 2015) a cycle time model to determine the performance of shuttles in the SBS/RS. Another approach with a single Markov queue $M|G|1$ was made by Karnig and Oser (2014). Eder developed in his three publications (Eder & Kartnig, 2016a, 2016b, 2017) an approach with a single open queueing system with limited capacity (SQLC). In these three publications, the interarrival time of the lifts and the service time of the shuttles are determined according to the reference point method of the VDI 3561 (1973). Table 1 gives an overview of the publication on SBS/RS.

Table 1. Literature overview.

Paper	Type of system	Type of cycle ^a	Model ^b	Storage depth
Heragu et al. (2011)	Tier-captive	SC	OQN	Single deep
Roy and Krishnamurthy (2011)	Tier-captive	SC	SOQN	Single deep
Marchet et al. (2012)	Tier-captive	SC	OQN	Single deep
Karnig and Oser (2014)	Tier-captive	DC	SQ	Single deep
Lerher et al. (2015)	Tier-captive	SC/DC	CTM	Single deep
Wang et al. (2016)	Tier-captive	SC/DC	OQN	Single deep
Lerher (2016)	Tier-captive	SC/DC	CTM	Double deep
Eder and Kartnig (2016a)	Tier-captive	SC/DC	SQLC	Double deep
Eder and Kartnig (2016b)	Tier-captive	SC/DC	SQLC	Single deep
Eder and Kartnig (2017)	Tier-captive	SC/DC	SQLC	Single deep

^aSC: single command cycle; DC: dual command cycle.

^bCTM: cycle time model; OQN: open queueing network; SOQN: semi-open queueing network; SQ: single queue; SQLC: single queue with limited capacity.

Table 2. Notation of the tier-captive single-aisle AVS/RS.

Δx	Distance between two storage slots
Δy	Distance between two tiers
λ	Poisson arrival rate (Smith, 2004)
ϑ	Mean throughput rate (Smith, 2004)
ϑ_{aisle}	Throughput of a single aisle
ϑ_{tier}	Throughput of a single tier
ρ	Utilization rate of the shuttle
A	Number of aisles
a_{lift}	Lift acceleration/deceleration rate
$a_{shuttle}$	Shuttle acceleration/deceleration rate
K	Capacity of the queueing system
$l_{I/O}$	Vertical distance between the first tier and the I/O point
n_{buf}	Number of buffers on each side of the aisle per tier
n_{slot}	Number of slots on each side of the aisle per tier
n_{tier}	Number of tiers
ρ_0	Probability of emptiness of a queueing system (Smith, 2004)
ρ_k	Blocking probability of a queueing system (Smith, 2004)
t_A	Interarrival time to a tier
t_{tL}	Time to transfer a tote to and from the lift
t_{tS}	Time to transfer a tote to and from the shuttle
t_{lift}	Cycle time of a lift for a single command cycle
t_{R_L}	Time to travel of a lift at single command cycle
$t_{R_{S_SC}}$	Time to travel of a shuttle at single command cycle
$t_{R_{S_DC}}$	Time to travel of a shuttle at dual command cycle
t_S	Service time of a shuttle
$t_{S_{SC}}$	Service time of a shuttle at single command cycle
$t_{S_{DC}}$	Service time of a shuttle at dual command cycle
$t_{shuttle_{SC}}$	Cycle time of a shuttle for a single command cycle
$t_{shuttle_{DC}}$	Cycle time of a shuttle for a dual command cycle
s	Coefficient of variation of the cycle times
v_{lift}	Lift velocity
$v_{shuttle}$	Shuttle velocity

This literature overview shows that tier-captive SBS/RS can be evaluated by using two mainly different queueing approaches. The first possibility is to model an open queueing network, the other option is to model an open QSLC. For both methods performance measures are needed, such as the utilization rate of the shuttles or cycle times of lifts and shuttles. To determine the cycle times, in some publications approximate continuous distributions, e.g. Eder and Kartnig (2016a, 2016b, 2017), are used. This generates an estimation error because of the different speed profiles depending on the distances to ride, that are not mentioned in these approaches. The limitation

of the open queueing network, from e.g. Marchet et al. (2012), Wang et al. (2016), is that the presented approach is able to calculate the waiting times within one handling cycle, but the maximum throughput can not be derived from this approach. In order to approximate the real system as good as possible, time continuous and spatial discrete models are used in this publication to calculate the cycle times. To consider the interactions between lifts and shuttles, an open QSLC is used. The resulting research gap of the literature review is the mathematical representation of the interaction of the subsystems lifts and shuttles and the effect on the throughput with spatial discrete and time continuous approach.

3. System description

The system that is investigated in this paper is a tier-captive single-aisle SBS/RS as shown in Figure 1. In front of each rack, there is a lift located that transports the totes from the I/O point to the tier and back. The I/O points are basically located on the first tier in front of the lifts. In each tier, there are buffers located between lifts and shuttles. Each shuttle is assigned to one tier in one aisle, which means that there are as many vehicles as tiers in a rack. Furthermore, these vehicles can handle one tote at a time along the aisle. The racks are single-deep, double sided and each storage location can hold one tote. The assumptions made here are similar to a SBS/RS produced by an European material handling provider. These assumptions are also been made in other

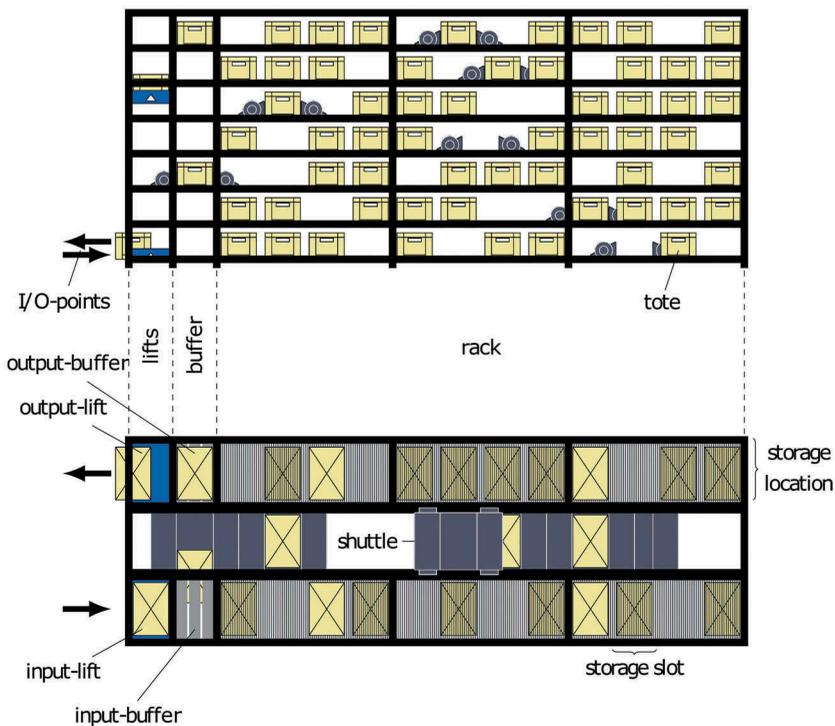


Figure 1. Shuttle-system.

publications of Eder and Kartnig (2016a, 2016b, 2017); Lerher (2016); Lerher et al. (2015), what assumption is made in which publication is depict at each point below.

The main assumptions and notations are listed below:

- Both lifts serve the transactions in single command cycle under an FCFS rule, so one lift is for the input and one is for the output (Eder & Kartnig, 2016a, 2016b, 2017).
- The shuttles serve the transactions in single and double cycle under an FCFS rule (Eder & Kartnig, 2016a, 2016b, 2017; Lerher, 2016; Lerher et al., 2015).
- The dwell point of the input lift is the I/O point (Eder & Kartnig, 2016a, 2016b, 2017).
- The dwell point of the output lift is the point of service completion (Eder & Kartnig, 2016a, 2016b, 2017).
- The dwell point of the shuttle is the point of service completion (Eder & Kartnig, 2016a, 2016b, 2017; Lerher, 2016; Lerher et al., 2015).
- The lifts and shuttles accelerate/decelerate with constant acceleration/deceleration (Eder & Kartnig, 2016a, 2016b, 2017; Lerher, 2016; Lerher et al., 2015).
- There is no difference in time between the transfer of totes to or from the lifts (Lerher, 2016; Lerher et al., 2015).
- There are always totes waiting on the I/O point to be stored (Eder & Kartnig, 2016a, 2016b, 2017).
- The order of the totes that need be re-stored next is evenly distributed among all stored totes (Eder & Kartnig, 2016a, 2016b, 2017).

4. Analytical approach

To determine the performance of an SBS/RS system, one aisle shall be modelled. Because of the fact that the storage and retrieval transactions are evenly distributed among all aisles and tiers, the performance of the system can be evaluated by modelling just one aisle (Epp, Wiedemann, & Furmans, 2017; Heragu et al., 2011; Marchet et al., 2012)

In storage-systems there are two main processes, the storage-process and the retrieval process. These two processes work the same way, only the direction is different. Because of this, only one process is modelled to determine the throughput. The process which will be treated to determine the throughput is the storage process of one tier in a single aisle. In this process, there are three main components that are important for the performance: the lift, the shuttle and the buffers between lift and shuttle. With these three components an open queueing model with limited capacity can be built. The process is shown in Figure 3. The appropriate queueing model for such a system is $M | G | 1 | K$. The first part (M) in this expression indicates the distribution of the interarrival times, the second part (G) is for the distribution of the service times. The number 1 in the expression indicates that there is only one service station (= one shuttle) per tier. The last part stands for the capacity of the queueing model, which is the capacity of the buffer plus the capacity of the shuttle. In our case $K = 1 + 1 = 2$. The notations used in this analytical approach are listed in Table 2.

The procedure of this approach is (see also Figure 2):

- (1) Determination of the interarrival time of the totes in each storage level by the lifts.
- (2) Determination of the service time within one storage level of the shuttles.
- (3) Determination of the throughput by using the open queueing model $M | G | 1 | K$.

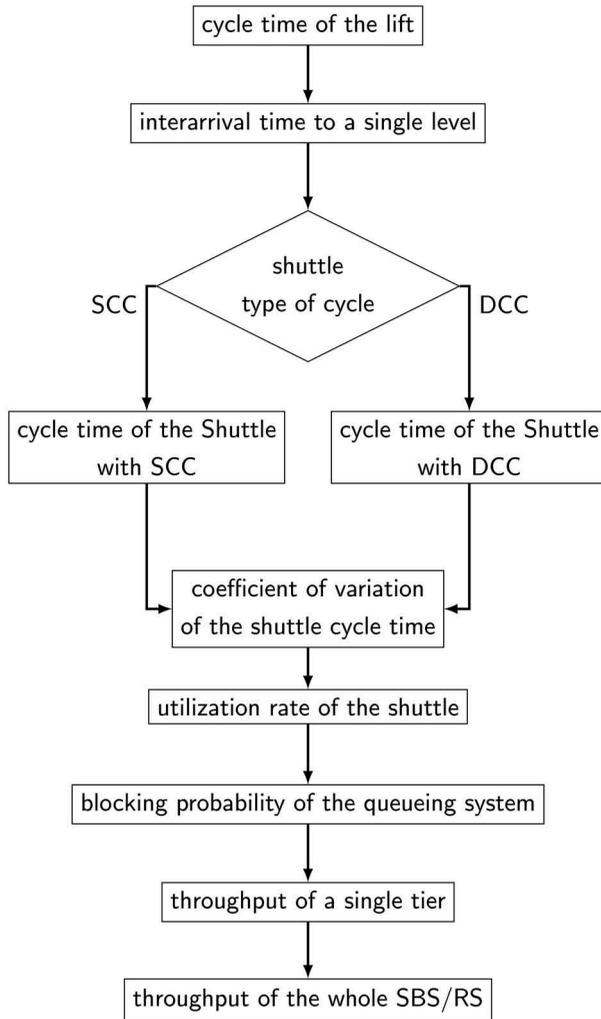


Figure 2. Flow chart to the procedure of this approach.

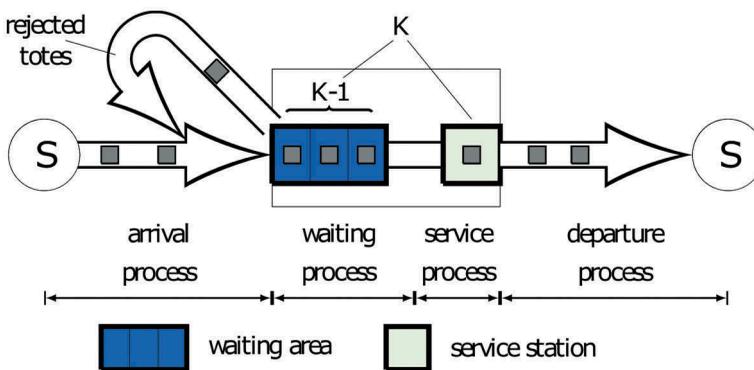


Figure 3. Open queueing model with limited capacity.

4.1. Interarrival time

The first thing needed for the calculation of the throughput of one tier is the interarrival time determined by the lifts. Therefore, the time for the ride and the times for transferring the totes to and from the lift is needed.

$$t_{lift} = 2 \cdot t_{R_L} + t_{t_L} \quad (1)$$

The mean time for the ride is:

$$t_{R_L} = \frac{1}{n_{tier}} \sum_{k=1}^{n_{tier}} \mathbf{t}(|l_{I/O} + (k-1) \cdot \Delta y|) \quad (2)$$

To consider the fact that within small distances the lifts will not reach their maximum speed, the function $\mathbf{t}(l)$ has to be split into two ranges. One range for distances less than $l < \frac{v_{lift}^2}{a}$:

$$\mathbf{t}(l) = 2 \sqrt{\frac{l}{a_{lift}}} \quad (3)$$

The other range for larger distances:

$$\mathbf{t}(l) = \frac{l}{v_{lift}} + \frac{v_{lift}}{a_{lift}} \quad (4)$$

Equation (1) delivers the cycle time of the lift for a single command lift cycle. The interarrival time is this cycle time times the number of tiers.

$$t_A = t_{lift} \cdot n_{tier} \quad (5)$$

This equation considers the fact that one lift serves all tiers of an aisle.

4.2. Service time for a single deep rack

The second thing that is needed for the determination of the throughput is the service time of the shuttles. This determination contains the same arguments as the determination of the cycle times of the lifts. There are the times for riding from A to B and the times for transferring the totes to and from the shuttle.

For single command cycle the following equation is developed:

$$t_{shuttle_{SC}} = 2 \cdot t_{R_{S_{SC}}} + t_{t_S} \quad (6)$$

The mean time for the ride is due to:

$$t_{R_{S_{SC}}} = \frac{1}{n_{slot}} \sum_{k=1}^{n_{slot}} \mathbf{t}(k \cdot \Delta x) \quad (7)$$

To consider the different equations depending on the distances, Equations (3) and (4) have to be used again.

The service time of the shuttles is as follows:

$$t_{S_{SC}} = 2 \cdot t_{shuttle_{SC}} \quad (8)$$

The multiplier 2 in this equation is because of the fact that a dual handling cycle is the reference.

For the dual command cycle the following equation is developed:

$$t_{shuttle_{DC}} = 2 \cdot t_{R_{S_SC}} + t_{R_{S_DC}} + 2 \cdot t_{t_s} \tag{9}$$

The first term is the same as for single cycle. The second term stands for the ride between the slot where a tote is transferred from the shuttle and the slot, where the next tote, that shall be retrieved, is located.

The mean time for this is due to:

$$t_{R_{S_DC}} = \frac{1}{n_{slot}^2} \sum_{k=1}^{n_{slot}} \sum_{l=1}^{n_{slot}} \mathbf{t}(|(k \cdot \Delta x - l \cdot \Delta x)|) \tag{10}$$

The service time of the shuttles with double cycle is as follows:

$$t_{S_{DC}} = t_{shuttle_{DC}} \tag{11}$$

4.3. Open queueing model $M | G | 1 | K$

To evaluate the influence of the buffers and the influence of the interaction between lifts and shuttles, a time continuous open queueing model with limited capacity is used (Smith, 2004).

$$\vartheta = \lambda \cdot (1 - p_k) \tag{12}$$

With the help of this model the throughput ϑ_{tier} of one single tier can be calculated:

$$\vartheta_{tier} = \frac{1}{t_A} \cdot (1 - p_k) \tag{13}$$

$$\vartheta_{tier} = \frac{1}{t_S} \cdot (1 - p_0) \tag{14}$$

There are two methods to determine the throughput. The first method is by the interarrival time and the blocking probability (Equation (13)). The second one is by using the service time and the probability of emptiness of the queueing system (Equation (14)). Blocking probability means that the system is completely filled so that no tote can enter the system. Applied to a shuttle system this means that the lift has to wait, for an empty space in the input-buffer. The probability of emptiness means that the server has to wait, because there is no tote in the queueing system. In a SBS/RS, this means that the shuttle has to wait for a tote.

The blocking probability of a queueing system is as follows (Smith, 2004):

$$p_k = \frac{\rho^{\frac{\sqrt{\bar{\rho} \cdot s^2 - \sqrt{\bar{\rho}} + 2K}}{2 + \sqrt{\bar{\rho} \cdot s^2 - \sqrt{\bar{\rho}}}} \cdot (\rho - 1)}}{\rho^{\frac{2 \cdot \sqrt{\bar{\rho} \cdot s^2 - \sqrt{\bar{\rho}} + K + 1}}{2 + \sqrt{\bar{\rho} \cdot s^2 - \sqrt{\bar{\rho}}}}} - 1} \tag{15}$$

This equation looks quiet complicated, but it contains only three arguments. The main argument is the utilization rate of the service station (= shuttle). This rate is given by the following equation, which contains the interarrival time defined by the lift and the service time of the shuttle:

$$\rho = \frac{t_S}{t_A} \tag{16}$$

Another argument is K . That is the capacity of the queueing system. It is the number of buffer spaces plus the capacity of the shuttles.

$$K = n_{buf} + 1 \tag{17}$$

The third argument is s , the coefficient of variation of the service time distribution. This coefficient is easy to calculate if the simplification is made, that there is only one equation for all distances (Equation (4)). Then the coefficient of variation for shuttles that make single handling cycles can be calculated as follows (Eder & Kartnig, 2017):

$$s = \frac{\sqrt{\frac{2 \cdot \frac{n_{slot} \cdot \Delta x}{v_{shuttle}}}{12}}}{t_S} \tag{18}$$

and for double handling cycle it is as follows (Eder & Kartnig, 2017) calculation:

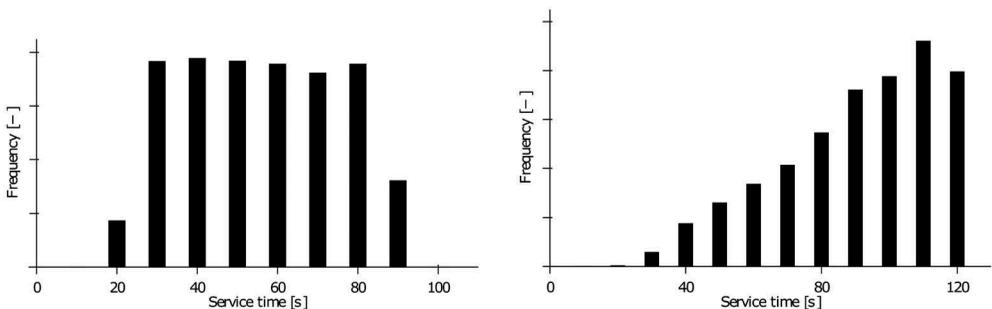
$$s = \frac{\sqrt{\frac{2 \cdot \frac{n_{slot} \cdot \Delta x}{v_{shuttle}}}{18}}}{t_S} \tag{19}$$

These simple equations can be used, because the service times for single cycles are evenly distributed (see Figure 4a) and the service times for double cycles are represented by a right skew triangular distribution (see Figure 4b). To prove this the results of a DES of the treated shuttle-system are shown in Figure 4.

The probability of emptiness of the queueing system contains the same arguments as the blocking probability and can be calculated as follows:

$$p_0 = \frac{\rho - 1}{\rho \frac{2 \cdot \sqrt{\hat{\rho} \cdot s^2 - \sqrt{\hat{\rho} + K + 1}}}{2 + \sqrt{\hat{\rho} \cdot s^2 - \sqrt{\hat{\rho}}} - 1} - 1} \tag{20}$$

The interarrival time distribution for one tier is shown in Figure 5. It can be seen that this distribution is a Poisson distribution that is expected for the M|G|1|K. The results shown in this figure are of a DES of the treated SBS/RS.



(a) Service time distribution for shuttles operating single command cycle

(b) Service time distribution for shuttles operating dual command cycle.

Figure 4. Examples of the service time distribution by the shuttles in one tier of a SBS/RS.

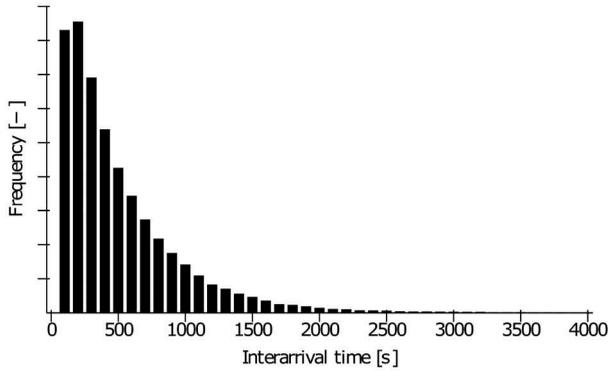


Figure 5. Interarrival time distribution for one tier.

The throughput of an aisle is equal the throughput of one tier multiplied by the number of tiers:

$$\vartheta_{aisle} = \vartheta_{tier} \cdot n_{tier} \quad (21)$$

5. Numerical study

The described analytical approach was validated by comparing it with a DES. This comparison is shown in Section 5.1. Section 5.2 will outline how to use the model to optimize an SBS/RS meeting the given requirements.

5.1. Numerical evaluation of the approximation quality

At the design process of an SBS/RS it is important to know the performance of one aisle to design the other parts of a storage system, such as the following conveyor. The knowledge of the influence of different design parameters helps to find an economically and ecologically ideal design of an SBS/RS.

To depict a broad variety of different settings, specific parameter configurations are chosen, Table 3 shows these parameters. The treated system has up to 50 tiers and up to 200 storage slots on each side of the aisle. The parameters in Table 3 were given by a European material handling provider of SBS/RS.

Table 3. Tested parameter configurations of the tier-captive single-aisle SBS/RS.

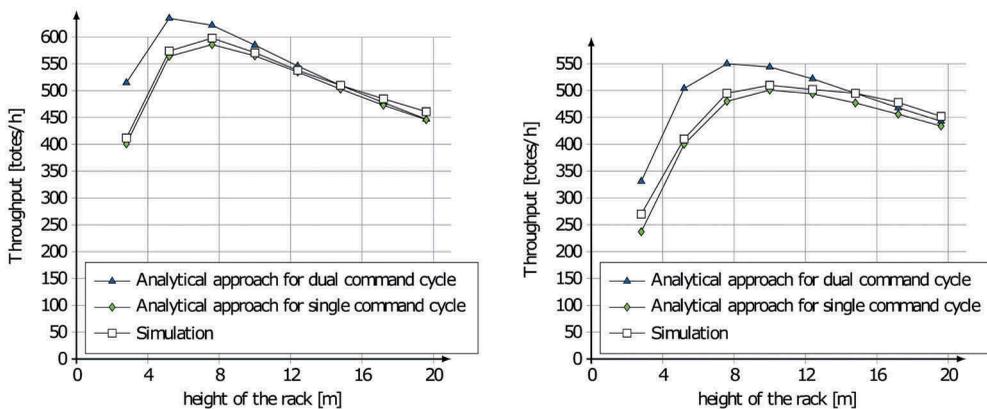
Parameter	Value
Number of tiers	$n_{tier} \in \{8, 14, 20, 26, 32, 38, 44, 50\}$
Number of storage slots on each side of the aisle per tier	$n_{slot} \in \{100, 200\}$
Number of buffers on each side of the aisle per tier	$n_{buf} = 1$
Distance between two storage slots	$\Delta x = 0, 5m$
Distance between two tiers	$\Delta y = 0, 4m$
Lift velocity	$v_{lift} = 5 \frac{m}{s}$
Lift acceleration/deceleration rate	$a_{lift} = 7 \frac{m}{s^2}$
Time to transfer a tote to and from the lift	$t_{t_l} = 2.8s$
Shuttle velocity	$v_{shuttle} = 2 \frac{m}{s}$
Shuttle acceleration/deceleration rate	$a_{shuttle} = 2 \frac{m}{s^2}$
Time to transfer a tote to and from the shuttle	$t_{t_s} = 8.4s$

For the validation the results of the analytical model were compared with the ensemble of 30 independent replications of the calculation with the simulation model. At each replication 10,000 totes were stored and retrieved. The simulation runs are performed by using the DES software SIMIO (version 10). The ordered totes are selected randomly. One storage and retrieval transaction consist of the following steps.

- (1) Lift travels to the I/O point.
- (2) Tote is transferred from the I/O point to the lift.
- (3) Lift travels to the destination tier.
- (4) Tote is transferred from the lift to the storage buffer.
- (5) Shuttle travels to the input-buffer.
- (6) Tote is transferred from the buffer to the shuttle.
- (7) Shuttle travels to the storage position.
- (8) Tote is transferred from the shuttle to the storage rack.
- (9) Shuttle travels to the storage position where the ordered tote is situated.
- (10) Tote is transferred from the rack to the shuttle.
- (11) Shuttle travels to the output-buffer.
- (12) Ordered tote is transferred from the shuttle to the output-buffer.
- (13) Lift travels to the respective tier.
- (14) Tote is transferred from the buffer to the lift.
- (15) Lift travels to the I/O point. 16. Tote is transferred from the lift to the I/O point.

The analytical approach is calculation according to Equations (1)–(20). The parameters used in this equation are shown in Table 3. Figure 6 shows the comparison between the results of the analytical model and of the simulation.

The curve of the results of the DES is exactly between the two curves of the results of the analytical model, this is shown in Figure 6. At the DES as well as in reality the shuttles operate a combination of single command cycle and dual command cycle, so the curve of the DES should be between these two curves of the analytical calculation.



(a) Throughput of a rack with 50m length and single deep storage. (b) Throughput of a rack with 100m length and single deep storage.

Figure 6. Comparison of the results of the analytical approach and the results of the discrete event simulation.

5.2. Optimization example

The described analytical approach allows a throughput optimization. The optimization example is based on the requirements given in Table 4.

The system has a storage capacity of 25,000 storage positions per aisle and there are no limitations of height and length. For the footprint the space needed for buffers, lifts and I/O points in front of the aisle is not considered. According to Epp et al. (2017), Heragu et al. (2011), Marchet et al. (2012), the throughput of multiple aisle SBS/RS can be calculated by knowing the performance of one aisle.

The parameters of lifts and shuttles assume the same values as in Section 5.1.

The results of this calculation are shown in Table 5. The ratio of $\frac{n_{tier}}{n_{slot}}$ is chosen for highest throughput by numerical differentiation of the developed analytical approach. In Figure 7, the throughput of one single aisle is shown depending on the number of tier with a given capacity of 25,000 storage locations. For more than one aisle the results of Table 5 are visualised in Figure 8. Similar to previous studies, e.g. Eder and Kartnig (2016a, 2016b, 2017) the optimal geometry for the maximum throughput depends on the number of aisles. Other publications, e.g. Lerher (2016),

Table 4. Requirements for the tier-captive SBS/RS.

Parameter	Value
Storage capacity	$N = 25,000$
Number of aisles	$A \in \{1, 2, 3, 4, 5\}$

Table 5. Optimization example.

A	n_{tier}	n_{slot}	N	Footprint [m^2]	ϑ_{aisle} [$\frac{1}{h}$]	ϑ_{system} [$\frac{1}{h}$]	Costs [e]
1	40	313	25,040	375.6	448	448	144,000
2	36	174	25,056	417.6	509	1018	152,000
3	27	155	25,110	558	559	1677	160,000
4	21	149	25,032	715.2	586	2344	172,000
5	20	125	25,000	750	608	3040	183,000

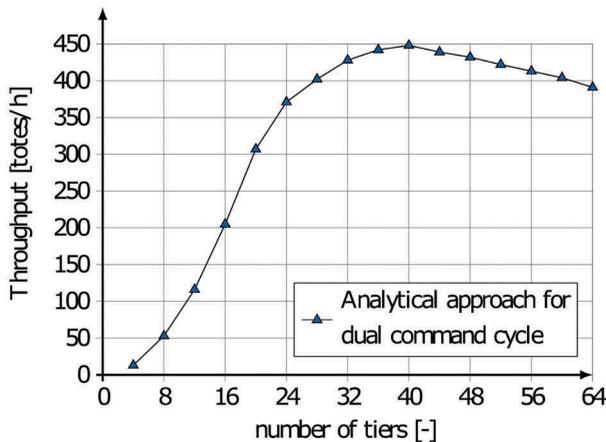


Figure 7. Optimization example of an aisle with a storage capacity of 25,000 storage locations.

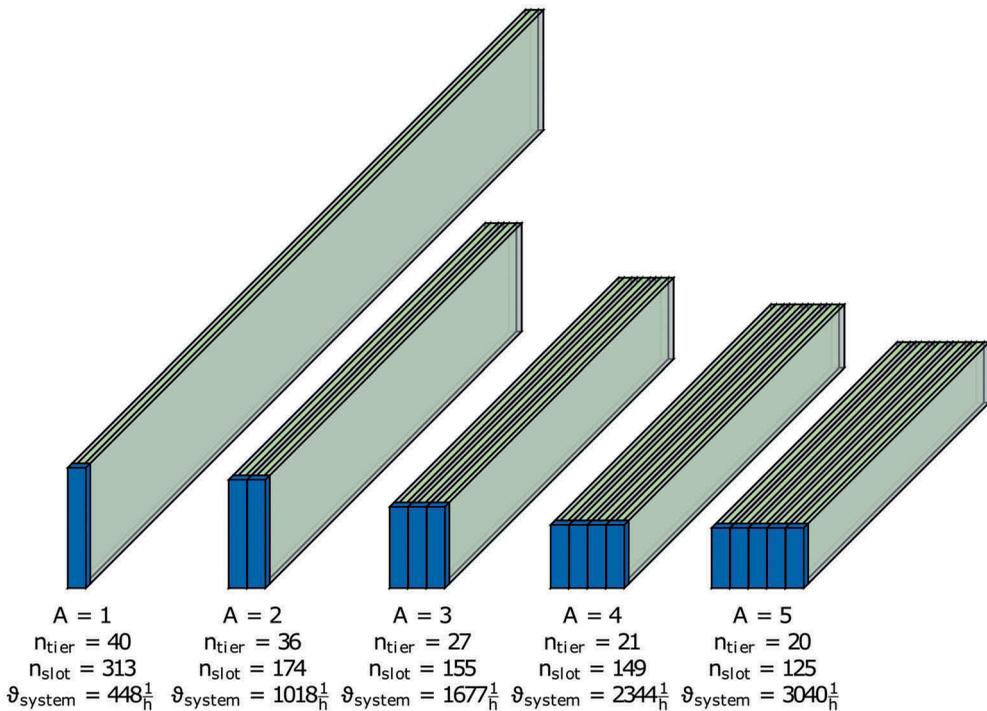


Figure 8. Optimization example.

Marchet et al. (2012), Wang et al. (2016), did not present any optimizations of the geometry. The given requirement with at least 25,000 storage locations leads to the fact that the length of the rack decreases with an increasing number of aisles. The number of tiers for maximum throughput decreases with increasing number of aisles. The footprint of the SBS/RS increases with increasing number of aisles. The throughput of the whole system ($\vartheta_{\text{system}}$) increases too with increasing of the number of aisles.

Due to the fact that all configurations have approximately the same number of storage positions, the costs of the SBS/RS mainly depend on the annual costs of the footprint, as well as the annualized costs of lifts and shuttles (see also Marchet et al. (2013)). To compare the different configurations of the example, the same cost structure as in Marchet et al. (2013) is assumed (i.e. $50e$ per m^2 footprint and year, 10 years of service, 10% interest rate, investment costs of $10,000e$ per shuttle, $50,000e$ per lift and $30e$ per storage position). The results of using these parameters on the determined data lead to increasing costs with an increase of the number of aisles. On the other hand, the increase of the throughput is far greater than the increase in costs. For example, switching from one to two aisles will increase the costs from $144,000e$ to $152,000e$, which is an increase of 5%. However the throughput increases for the same modification is from $448\frac{1}{h}$ to $1018\frac{1}{h}$ what means an increase of around 127%. This leads to a decision tool on how to design a SBS/RS to meet the requirements with a minimum on cost.

6. Conclusion

Tier-captive SBS/RS are being used more and more in many industries due to their high system performance. However, few analytical decision tools are available to assess the performance of these SBS/RS. There are two ways to estimate the interactions between shuttles and lifts. One uses an open QSLC to estimate the interactions between lifts and shuttle. The second one uses open queueing networks to treat the interactions between shuttles and lifts. These open queueing network approaches have the limitation that they only calculate the waiting times between lifts and shuttles within one transaction and it is not possible to deduce the throughput of a whole aisle out of this calculation.

Consequently, we present a method for calculation of the performance measures of tier-captive single aisle SBS/RS that is easy to handle and provides practicable results. The system is modelled as a space discrete continuous time open QSLC. The interarrival times and service times are evaluated by a discrete spatial values cycle time model of lifts and shuttles. The accuracy of the analytical model in comparison to a DES is validated.

The storage process is modelled by an open queueing model with limited capacity, which is the best approximation of the reality. Also the real distributions of the interarrival times and service times were used in this analytical model. In general the presented approach reaches a high approximation quality. Furthermore, the parameters used for the validation of the analytical approach are given by an European material handling provider. This approach assumes a control of the entire system which seeks to minimize the waiting times between the subsystems lifts and shuttles, by looking ahead to further retrieval operations. Another limitation is that this approach does not mention any influences on the way before and after an storage aisle.

Finally, it is outlined how the presented queueing model can be used to optimize tier captive SBS/RS for the given requirements, such as storage capacity, throughput, height and length. Also the resulting costs of the design are calculated here. The example shows the influence of the number of aisles in the system. This approach can be used to design a storage system for given requirements on the system, this is an advantage for a provider of such SBS/RS. For example, when designing a new storage system, this approach can be used to calculate the key data for building such a system, in a very simple and accurate way. Although it is possible to find the limitation factor of an existing SBS/RS and with this knowledge it is possible to improve this bottleneck. The assumptions made in this paper are similar to a SBS/RS of an European material handling provider. Also a comparison of the results of the presented analytical approach to the throughput of an existing SBS/RS shows that this approach gives accurate results.

Further work will be dedicated to the SBS/RS with alternative system configurations e.g. different number of handling devices, different storage management policies like as class based storage policies, different policies e.g. lifts operation in double cycles and different storage depths.

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