A Modified Gomory-Hu Algorithm with DWDM-Oriented Technology

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Abstract. Optimization of the topology of computer networks based on the classical Gomory-Hu algorithm does not take the specific transfer technology into account. For WDM technology requirements this leads to a redundancy of channel capacities.

To reduce the redundancy of allocating network resources, we propose a modification of the Gomory-Hu algorithm which takes account of the specifics of DWDM technology – not at the final stage but already at intermediate stages in the process. The original algorithm proposed by Gomory and Hu involves the decomposition of the graph of the input network into ring subnets of different dimensions. Our modified algorithm takes account of the technical parameters of the DWDM technology for each ring during the decomposition.

We illustrate our method by an example. The technique can be extended to large networks, which may lead to a significant economic effect.

Keywords: Network topology · Channel capacity · Dense Wavelength Division Multiplexing (DWDM) · Gomory-Hu algorithm

1 Introduction

Current developments in the fields of data transmission technology are characterized by intense growth of information flow and increasing requirements for

Supported by the Austrian Federal Ministry of Education, Science and Research and the Ukrainian Ministry of Science and Education, under the joint Ukraine-Austria R & D project 'Traffic and telecommunication networks modelling', project No. UA 10/2017/0118U001750, M-130/2018.

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I. Lirkov and S. Margenov (Eds.): LSSC 2019, LNCS 11958, pp. 547–554, 2020.
https://doi.org/10.1007/978-3-030-41032-2_63

the quality of transmission. New technologies for various purposes and their further exploitation require significant costs for modern telecommunication equipment and channel resources. Thus, the problem of optimization of data transmission networks, the hardware basis of which are backbone networks, is becoming increasingly important.

Remark 1. The algorithm proposed in this paper has not yet been implemented in software. In the context of the joint Ukraine-Austria R & D project 'Traffic and telecommunication networks modelling', we are preparing an implementation in Maple[1] using the GraphTheory package. As a further step, following ideas from [1], parallelization of the process can be envisaged based on a master/slave model which is convenient to use for prototyping purposes.

2 Problem Setting

One of the major ways to ensure cost-efficient use of Wide and Metropolitan Area Networks relies on optimization of their topology. Many works have been devoted to the design of topologies of main networks, examples of which can be found in [5,6], where different approaches for the choice of network topology are considered and methods and algorithms for optimization are analyzed and systematized.

The cost of using networks is mainly determined by occupying channel resources, which significantly depend on the network capacity. The principles of the Gomory-Hu topological design are described in [2,3,6]. Gomory and Hu proposed an algorithm which enables to provide a synthesis of network topology and choice of channel capacities. The network designed according to this algorithm enables transmission of a maximum given input flow with a minimum required total capacity over the channels. In [9] a simulation model based on the Gomory-Hu algorithm (besides some other algorithms) was implemented. In [11], devoted to the embedding of virtual topologies in network clouds, the algorithmic steps proposed start with building the Gomory-Hu tree.

Optimization via the Gomory-Hu algorithm results in channel capacities which do not take account of particular transmission technologies in the different channels. However, the capacity of the separate channels should be selected in accordance with the requirements of the transfer technology used. Taking into account different capacities of the channels it is to be expected that a modified optimization procedure will lead to an improved performance with the available network resources.

A modified Gomory-Hu-type algorithm for the optimization of networks using SDH (Synchrone Digital Hierarchy) technology was proposed in [7]. The purpose of the present paper is to investigate the possibility of minimizing the excess capacity by modifying the Gomory-Hu algorithm in a way taking account of the available channel capacities in accordance with the requirements of the DWDM (Dense Wavelength Division Multiplexing) technology.

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[1] Maple is a product by Maplesoft™ (www.maplesoft.com).

As a state of the art, the DWDM transport technology provides the highest speed among all other technologies for transmitting data via an optical pair. High speed is provided by *wavelength multiplexing*, where dozens or even hundreds of independent data flows are transmitted through a single optical pair, different flows being transmitted in different optical ranges.

Current commercial DWDM systems use up to 80 transfer channels of capacity up to 40 Gbps, which amounts to a total transfer capacity up to 3.2 Tbps. These systems are able to overcome distances of more than 2000 km without signal renewal [4]. Current experiments enable transferring an even higher flow, but only for distances of a few hundreds km [8,10]. Recently, a DWDM with 200 transfer channels with a capacity of 40 Gbps per channel was realized, resulting in a total capacity of 8 Tbps per fiber [4]. An example of a system in which each channel carries 100 Gbps and 192 channels per fiber pair, which amounts to a total of 19.2 Tbps per pair, is reported in [12].

3 The Classical Gomory-Hu Algorithm

In the following we first review the details of the classical Gomory-Hu algorithm. The input data is a set of nodes, together with the requirement of exchange of information and the intensity of flows that need to be provided between them. These data are represented in form of an undirected weighted graph, where the weights of the edges represent the flows to be transmitted.

The algorithm is based on certain rules for splitting the given graph into several subnets, with the goal of performance optimization as explained below. All these subnets, except the last one, are weighted ring subnets with constant weight. The last subnet is a direct channel between two nodes or it may be void.

The result of the resulting optimization procedure is again a graph representing the topology of the network after optimization. The resulting weights represent the required channel capacities after optimization. In this way one can find the topology of the network and the capacity of its channels for which transmission with maximum flow is ensured, and at the same time the weights of all edges (i.e., the required capacity of the communication channels) will be minimal.

By superposition of all the resulting subnets one obtains an *optimized network that will feature minimum total capacities of the channels (edges), while providing transmission of maximum flow.*

The Gomory-Hu algorithm consists of two major stages:

(i) decomposition,
(ii) superposition.

ad (i): Decomposition

1. Specification of the input network in form of a weighted non-directed graph $A := G_{\text{in}}$. The weights of the edges represent the required intensities of flows.
2. Decomposition of the graph $A = \{a_{ij}\}$ into

– a ring graph[2] SNk which includes all the nodes of the graph A, and assigning to each edge of the ring the weight $W_{\min}/2$, where k is the cycle number and $W_{\min}$ is the minimal weight of the edges of the graph A;
– a graph B which is obtained by subtracting the value $W_{\min}$ from the weight of each edge of graph A whose weight is greater than zero.

If the number of edges in the resulting graph B is greater than one, then we accept $A = B$ and go back to step 2, repeating the iteration for creating a cycle. Otherwise the decomposition stage is completed. Then, the graph B will consist of one segment with a weight greater than zero, or it may be void.

ad (ii): Superposition

Construction of the output graph by integration of all graphs in which the input graph was decomposed in step **(i)**.

Example 1. Fig. 1 shows the results for the following input data:

$$\begin{aligned} a_{12} &= 1000 \text{ Gbps}, \quad a_{13} = 800 \text{ Gbps}, \quad a_{15} = 800 \text{ Gbps}, \\ a_{24} &= \ \ 300 \text{ Gbps}, \quad a_{25} = 500 \text{ Gbps}, \quad a_{45} = 500 \text{ Gbps}. \end{aligned}$$

Let us discuss these results. The verification is performed for maximum flow, in our case it is $a_{12} = 1000$ Gbps. In the resulting network, this flow can be transmitted simultaneously in the following way:

$$\begin{aligned} a_{12} &\Rightarrow 600 \text{ Gbps}, \\ a_{15432} &\Rightarrow 250 \text{ Gbps}, \\ a_{1532} &\Rightarrow 150 \text{ Gbps}, \end{aligned}$$

where a_{15432} corresponds to the path from node 1 to node 2 across the nodes $5, 4, 3$, and a_{1532} to the path from node 1 to node 2 across the nodes $5, 3$. The sum of these flows is 1000 Gbps. Thus, the network is able to transmit the maximum flow without any excess resources, i.e., all network resources are busy for transmission of maximum flow. The total capacity of all channels is 2050 Gbps.

When constructing a network or renting a channel, real channels will be selected taking into account the capabilities of the DWDM technology. Let us denote by C_{ij} the throughput of the channel between nodes i and j. According to the optimized results obtained by the Gomory-Hu algorithm for Example 1 (see Fig. 1), we need channels with capacities

$$\begin{aligned} & C_{12} = 600 \text{ Gbps}, \quad C_{23} = C_{15} = 400 \text{ Gbps}, \quad C_{53} = 150 \text{ Gbps}, \\ \text{and} \quad & C_{54} = C_{43} = 250 \text{ Gbps}. \end{aligned}$$

However, for this example we have assumed that the capacity of one optical fiber is 400 Gbps. This means that for further optimization the capacity of each

[2] SN is an abbreviation for 'SubNet'.

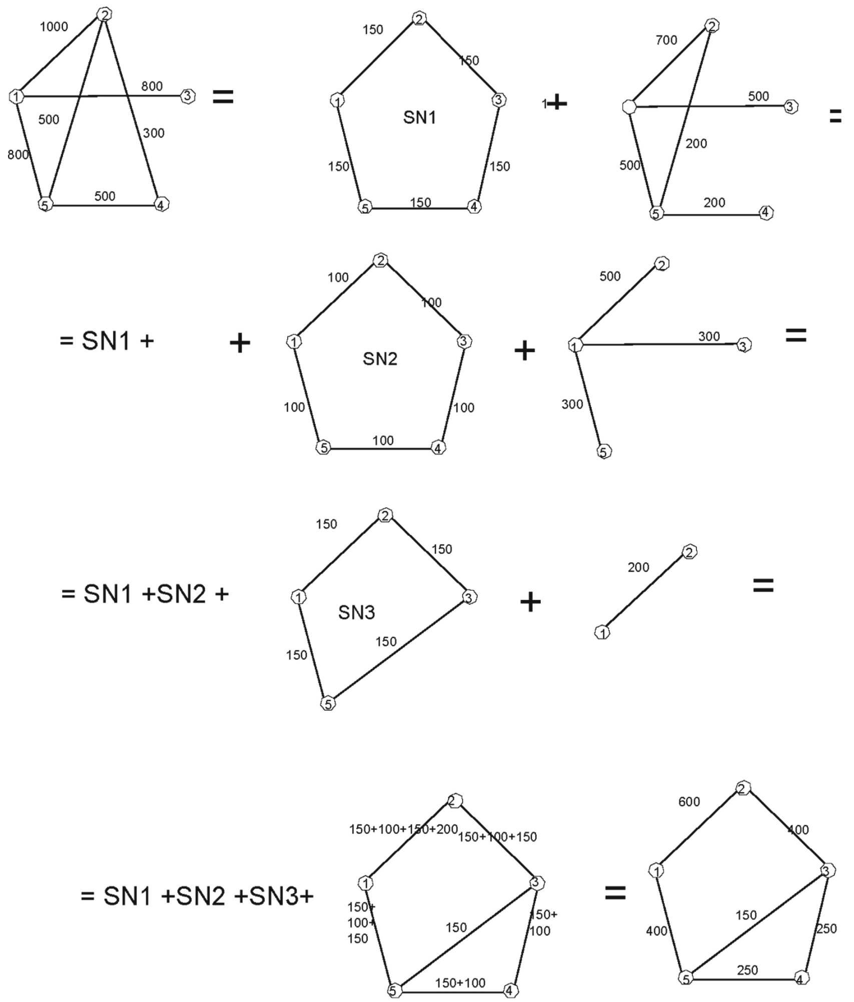

Fig. 1. Example 1: Illustration of the results obtained by the classical Gomory-Hu algorithm.

channel should be a multiple of 400 Gbps. Therefore, in Fig. 2, in the resulting network adapted to the DWDM technology, we increase some of the capacities:

$$C_{12} = 800 \text{ Gbps}, \quad C_{53} = C_{54} = C_{43} = 400 \,\text{Gbps}.$$

As a result, the total capacity and hence the cost is higher than for the version obtained by the classical Gomory-Hu algorithm.

Example 2. (based on Example 1): We now assume that the DWDM technology, in particular, the Transport Platform Cisco ONS 15808, is used. For extended-long-haul applications up to 2250 km the Cisco ONS 15808 supports 40 channels of 10 Gbps capacity. The core parameter of a DWDM system transfer is the capacity C of an optical fiber, given by $C = n\,V$, where, as before, n is the total number of channels of one fiber and V is the transfer speed of a single channel. This means that the capacity of one fiber is $C = 40 \cdot 10$ Gbps $= 400$ Gbps. Accordingly, the resulting network shown in Fig. 1, with its orientation on the Transport Platform Cisco ONS 15808 with transfer capacity equal to 400 Gbps per fiber pair, will now have the form presented in Fig. 2.

The total capacity of all channels has increased to 2800 Gbps, and therefore this method of adaptation to DWDM technology has generated redundancy of resources.

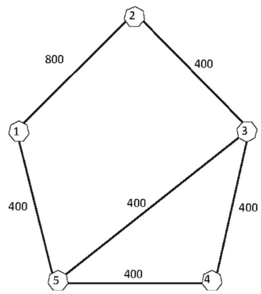

Fig. 2. Example 2: Network resulting from the Gomory-Hu algorithm adapted to the requirements of the Transport Platform Cisco ONS 15808.

4 A Modified Algorithm

To reduce the redundancy of channel capacities we now propose a modified Gomory-Hu algorithm. The basic idea is that adaptation to the DWDM technology is carried out already in the intermediate steps of the optimization algorithm, namely when creating ring subnets. If in the classical algorithm the channel capacity of ring subnets was chosen as $W_{\min}/2$, it is now replaced by the closest higher value $j\,C$, where C is the capacity per fiber. Obviously this also increases redundancy, but it may be taken into account in the next steps in the construction of new ring subnets. Their capacity will be chosen smaller taking into account the introduced redundancy and may be smaller compared to the classical Gomory-Hu algorithm.

The modified algorithm can be represented as follows.

1. Specification of the input network in form of a weighted non-directed graph $A := G_{\text{in}}$. The weights of the edges represent the required intensities of flows.
2. Find the minimal weight of the edges of the graph $A - W_{\min}$, and replace $W_{\min}$ by the closest higher value $j\,C$ for $j = 1, 2, 3, \ldots C$, where C is the capacity per fiber.

3. Decomposition of graph A into
   – a ring graph SNk which includes all the nodes of graph A, and assigning to each edge of the ring the weight $W_{\min}/2$ where k is the cycle number;
   – a graph B, which is obtained by subtracting the value of $W_{\min}$ from all edges of graph A whose weight is greater than zero, and substituting negative weights by zero.
4. If the number of edges in graph B is greater than one, then we accept $A = B$ and go back to step 2.
5. Replace the non-zero edges of graph B by the nearest higher value $j\,C$, $j = 1, 2, 3, \ldots$, where C is the capacity per fiber.
6. Integration of all graphs of SNk and graph B.

Example 3. (based on Example 1): An illustration of the proposed approach is shown in the example of the implementation of the modified algorithm for the same input data as before for the classical version. Fig. 3 shows the process and result of optimization based on the modified Gomory-Hu algorithm.

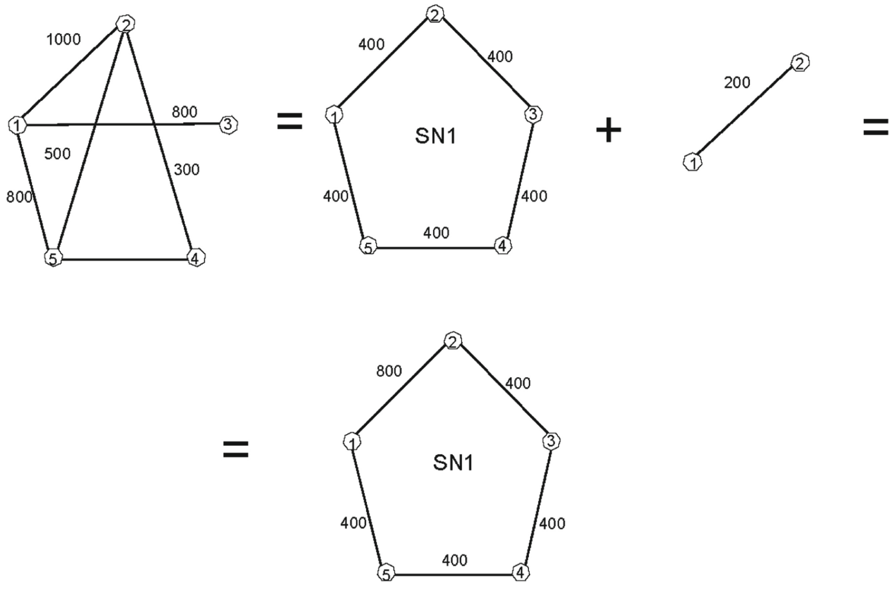

Fig. 3. Example 3: Illustration of the results obtained by the modified Gomory-Hu algorithm

The total required capacity for all channels amounts to 2400 Gbps. When using the classical algorithm with adaptation to DWDM technology, it was 2800 Gbps. So there is a redundancy reduction of 400 Gbps.

The gain in capacities, or its absence, in the transition from the classical algorithm to the modified one, will depend on the intensities of the input flows.

Since an a priori analysis of input data does generally not allow to predict which algorithm will yield better results, it is recommended for each input set to investigate both options (classical and modified), to compare the results and choose the option with minimal redundancy.

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