Implementation of a fuzzy model predictive controller for biomass combustion

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ABSTRACT: Implementing state of the art model based control algorithms to supervise and control nonlinear thermo-chemical processes significantly increases their performance. In combination with process feedback from measurements, predictive control algorithms can make use of the underlying models to provide restricted estimations of the likely future process behaviour. This information is beneficially used to provide smooth transient behaviour between different set points with respect to actuator saturations. In order to cover the entire operating range of the considered nonlinear system, multiple parallel, local linear predictive controller are set up. The data calculated by those individual controller is convoluted and weighted by membership functions according to a set of fuzzy-rules. The result is once again a nonlinear feedback-system, which can compensate local inaccuracies and considers future reference changes. In this work, a straight forward approach for the implementation of a fuzzy model predictive controller for a given nonlinear process model is shown.

1. INTRODUCTION

Standard control algorithms, like PID controller, are often used in industrial applications for small and medium scaled biomass combustion plants (100 kW – 5 MW). Their main advantage is that they are comparably simple to implement and benefit from lots of existing expert-knowledge. The configuration of these controller is realized in many cases for a nominal operating point and the control parameter are therefore not necessarily optimal for the entire range of operation. To overcome these issues, more advanced control strategies are being implemented, of which some make use of process models in their controller design.

One of these approaches is the model predictive controller (MPC) design. In comparison to other control algorithms, the MPC is able to use its internal dynamic model to make short-term predictions of the future process behaviour. This leads to a vast reduction of actuator efforts and brings the system in a predefined optimal way to a desired reference value. The natural limitation though to this ability is the quality of the internal model. Like all model based control algorithms, their general application can suffer from the potentially high effort to find a sufficiently accurate model for the Multi-Input-Multi-Output (MIMO) system.

A possible design process for MPC's is based on linear models. To overcome the typical drawbacks introduced by linearization, multiple MPC's are derived for different operating points and merged together with the support of fuzzy-logic. The fuzzy principle convinces due to its simple fundamental idea, in which linguistic rules and expressions are transformed into algebraic equations (Sugeno, 1993). The combination of these methods contributes to a beneficial integration into advanced energy systems, like smart grids or smart homes (Killian, 2014) and other industrial applications, while still being subject to local optimality criterions for a specific plant. In this work, an underlying nonlinear process model of a small scaled biomass grate combustion furnace provides the basis for such a network of local linear models, which are individually controlled by a local MPC. In order to efficiently partition the operating space of the furnace, the *nu-gap* metric is used to find the smallest number of submodels necessary to describe the process given a predefined quality requirement (Du, 2009).

The combination of the introduced methods will yield an advanced overall control-scheme, based on an existing biomass combustion model from literature. The implementation process of the fuzzy model predictive controller (FMPC) with beneficial use of the *nu-gap* metric will be shown and by investigating the results of this dynamic approach its performance for a small scaled biomass combustion furnace (operated around 100 kW) will be illustrated. This work is structured as follows: First, the basic nonlinear model will be reviewed and brought into a linear state space representation with the help of the *nu-gap* metric. Then, the MPC algorithm and the fuzzy formulation will be shortly introduced. Finally, the results will be presented and discussed.

2. BIOMASS COMBUSTION MODEL

The model used for the implementation of the FMPC is based on the work presented in (Placek, 2015). The set of nonlinear differential equations is derived from first principle modelling, which can be considered a standard approach for the description of biomass combustion systems with control focus, although sometimes the equations are extended by means of grey-box modelling. For more examples, see (Kortela, 2015) or (Paces, 2011). One of the biggest differences of the considered model compared to the above for example is, that there is no Oxygen equation presented, which is typically applied for controlling the combustion stoichiometry and therefore contains valuable process information about the burnout level and the emission production of the plant.

2.1 STATE SPACE REPRESENTATION

For the purposes of this work, three out of five available states have been chosen as controlled variables y. The states x are denoted as *temperature of the combustion chamber refractory* ϑ_r (K), *temperature of the hot flue gas* ϑ_g (K), *temperature of the heated water* ϑ_{wo} (K), *current mass on the grate* m_b (kg) and *temperature at the sack entrance* ϑ_e (K) respectively. The chosen outputs are $y_1 = \vartheta_{wo}$, $y_2 = \vartheta_g$ and $y_3 = \vartheta_e$. The system inputs u are the *fuel mass flow* $\dot{m}_{in}$, the *primary air supply* $\dot{m}_{pa}$ and the *secondary air supply* $\dot{m}_{sa}$ (all in kg/s) yielding the system vectors

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} \vartheta_r \\ \vartheta_g \\ \vartheta_{wo} \\ m_b \\ \vartheta_e \end{pmatrix}, \qquad u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \dot{m}_{in} \\ \dot{m}_{pa} \\ \dot{m}_{sa} \end{pmatrix} \qquad \text{and} \qquad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_2 \\ x_5 \end{pmatrix}.$$

The nonlinear continuous system can then be written as:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \tag{1}$$

The according state space representation after the linearization of (1) becomes

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + B_m u(t) \\ y(t) &= C_m x(t) + D_m u(t), \end{aligned} \tag{2}$$

with the linear time invariant (LTI) system matrices A_m as the 5x5 system matrix, B_m as the 5x3 input matrix, C_m as the 5x3 output matrix and D_m as the 5x3 direct input-output gain matrix. The index m indicates the number of considered operating points, which will be discussed in the next section.

2.2 SELECTION OF LINEARIZATION POINTS WITH THE NU-GAP METRIC

One way to find suitable points for the linearization of the nonlinear plant is to use experience or knowledge of the process. In this work, the *nu-gap* metric is considered as methodical approach to find a minimal number of linearization points that sufficiently describe the nonlinear plant. The *nu-gap* metric splits the partitioning space into n local linear models, where n is an arbitrary number that determines the grid size or resolution of the gap map. The n obtained transfer functions map the partitioning space and their distance is compared to each other in a gap sense. Once a predefined distance, which is equal to a quality threshold, is reached, a new model is created (Du, 2009).

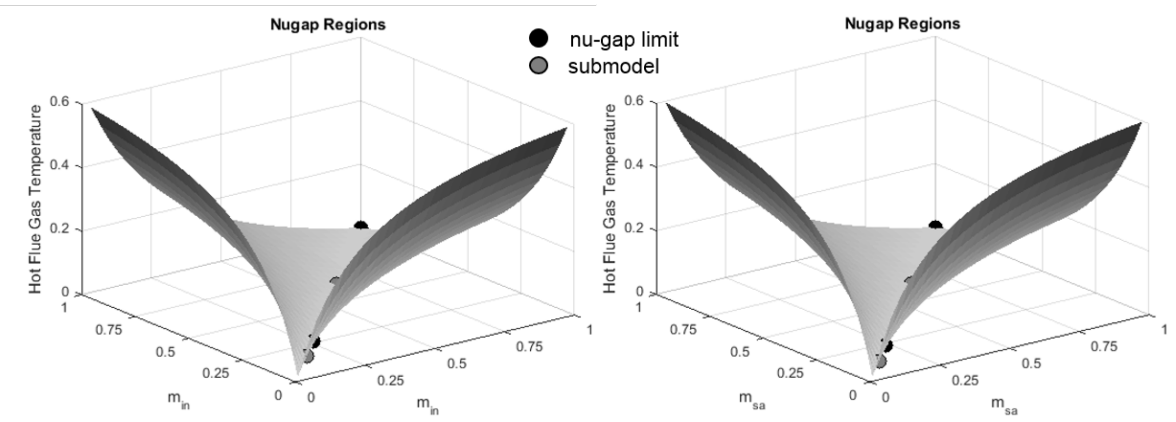

Figure 1: nu-gap map for the temperature depending on $\dot{m}_{in}$ (left) and $\dot{m}_{sa}$ (right)

For the determination of the operating points, the hot flue gas temperature ϑ_g is chosen as partitioning variable, since it indicates the behaviour of all other states well enough and responds the fastest to changes of the air supply. The maximal gap distance is set to 0.33, the grid size is n = 30 and the partitioning space ranges from 673 to 1273°K. The *nu-gap* metric yields a very similar map for ϑ_g with respect to the inputs $\dot{m}_{in}$ and $\dot{m}_{sa}$ over the entire operating range, which is depicted in figure (1). This leads to two submodels in total, located at the grid positions 5 and 19. The gap values obtained for the primary air are all below the quality threshold and therefore not critical for the choice of the submodels.

3. MODEL PREDICTIVE CONTROL

The MPC conducts an optimization task for every time step k of the discrete state space system in order to find the optimal solution for a given quadratic cost function. The results of the optimization task are the future inputs for the nonlinear plant based upon the underlying linear models and potential process and actuator constraints.

3.1 BASIC MPC-ALGORITHM WITH RECEDING HORIZON PRINCIPLE

The following formulations are based on the discrete state-space representation of the linearized model and on (Wang, 2009). After discretization, the MIMO system from (2) becomes

$$x_m(k+1) = A_m x_m(k) + B_m u(k)$$
$$y(k) = C_m x_m(k) \tag{3}$$

where it is implicitly assumed that due to the receding horizon principle the inputs cannot directly affect the outputs and therefore D_m is zero. The vector of future control increments ΔU and predicted outputs Y becomes

$$\Delta U = [\Delta u(k)\ \ \Delta u(k+1)\ \ \Delta u(k+2) \dots \Delta u(k+N_c-1)]^T$$
$$Y = [y(k+1|k)\ \ y(k+2|k)\ \ y(k+3|k) \dots y(k+N_p|k)]^T \tag{4}$$

with

$$\Delta u(k) = u(k) - u(k-1). \tag{5}$$

The time the algorithm is able to see into the future is denoted as prediction horizon N_p and the second characteristic value is denoted as control horizon N_c , which determines when the calculated inputs are to be utilized by the controller. The cost function for the optimization for ΔU then is written as

$$J(x(k), \Delta U) = \left(Y_{ref} - Y\right)^T Q \left(Y_{ref} - Y\right) + \Delta U^T R \Delta U, \tag{6}$$

where Y_{ref} is the block-vector of future reference values and R is the weighting matrix of the control increments, which can be used to influence the relative costs of the inputs. The optimal solution for the control signal ΔU is found by setting the first derivative of (6) to zero:

$$\frac{\partial J}{\partial \Delta U} = -2\emptyset^T \left(Y_{ref} - F(x(k))\right) + 2(\emptyset^T Q \emptyset + R)\Delta U = 0 \tag{7}$$

From the solution of (7), the optimal control increment can be found as

$$\Delta U(Y_{ref}, x(k)) = (\emptyset^T Q\ \emptyset)^{-1} \emptyset^T (Y_{ref} - Fx(k)), \tag{8}$$

where Q is the weighting matrix for control errors and the matrices $\emptyset$ and F have the following form:

$$F = \begin{pmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{Np-1} \end{pmatrix}; \qquad \emptyset = \begin{pmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ CA^2B & CAB & CB & \dots & 0 \\ & & \vdots & & \\ CA^{Np-1}B & CA^{Np-2}B & CA^{Np-3}B & \dots & CA^{Np-Nc}B \end{pmatrix};$$

Although N_p future time steps are concerned and N_c future control increments are being calculated, the receding horizon principle only implements the first control increment of ΔU and rejects all others. This procedure is repeated for all future time steps anew, leading to a permanently updated closed loop system that considers the process feedback and existing constraints in the optimization task. A balanced and problem related choice of N_p and N_c can separate a well-adjusted MPC from a poorly one, since solely increasing both definitely results in a higher calculation effort, but not necessarily in a better performance.

3.2 CONSTRAINT MPC

In order to respect physical bounds as well as actuator saturations, constraints are implemented into the control algorithm. In contradiction to problems caused by the integral part of a controller, where actuator saturations or fast changes of the set points can cause wind-up effects, the solution of the underlying optimization problem is able to consider these effects in every step. Formulating the specific limits as inequality constraints, the solution of (7) becomes subject to:

$$\begin{aligned} \Delta u^{min} &\leq \Delta u(k) \leq \Delta u^{max} \\ u^{min} &\leq u(k) \quad \leq u^{max} \\ y^{min} &\leq y(k) \quad \leq y^{max} \end{aligned}$$

If a feasible solution for ΔU exists that satisfies (7), it is considered optimal, because there is only one solution for the quadratic problem. The chosen constraints are listed in table (1).

Table 1: Constraints for the MPC's

	$\Delta u(k)$ in g/s			$u(k)$ in g/s			$y(k)$ in K		
input	$\dot{m}_{in}$	$\dot{m}_{pa}$	$\dot{m}_{sa}$	$\dot{m}_{in}$	$\dot{m}_{pa}$	$\dot{m}_{sa}$	y_1	y_2	y_3
max	0,10	0,17	0,33	15,00	25,00	50,00	373	1273	523
min	0,10	0,17	0,33	5,00	8,33	16,67	298	673	333

4. FUZZY SYSTEMS

Especially for highly nonlinear processes, the error introduced due to linearization may become very large, maybe even too large for a stable operation with only one controller, not to mention uncertainties introduced during modelling. The fuzzy formulation allows to convolute two or more MPC's for the investigated system to improve the performance over the entire operating range of the plant, merging the solution of all relevant controller according to the fuzzy rules and the desired set points.

4.1 FUZZY FORMULATION AND MEMBERSHIP FUNCTIONS

The fuzzy formulation is based on the work presented in (Nelles, 2013). Gaussians membership functions are chosen, because they are continuously differentiable and have another property, which will be explained at the end of this section. The membership functions are expressed as

$$\mu_i(u) = \exp\left(-\frac{1}{2}\left(\frac{(u_1-c_{i1})^2}{\sigma_{i1}^2}\right)+\cdots+\left(\frac{\left(u_p-c_{ip}\right)^2}{\sigma_{ip}^2}\right)\right) \qquad i = 1,2,\dots,m \tag{9}$$

where p is the number of inputs, m is the number of submodels, c are the center points of the partitioning variable, given by the operating points of ϑ_g, and σ is the spread or the width of the Gaussians, which can be used for tuning. The validity functions ϕ normalize the membership functions μ and are evaluated as

$$\phi_i(u) = \frac{\mu_i(u)}{\sum_{j=1}^{M}\mu_j(u)}, \tag{10}$$

such that

$$\sum_{i=1}^{m} \phi_i(u) = 1 \tag{11}$$

holds, which is important to ensure at all times. In this representation of the Gaussians, the fuzzy model is now equivalent to a Takagi-Sugeno fuzzy model. Finally, by using the validity functions the actual plant input becomes

$$u_{fmpc} = \sum_{i=1}^{m} u_i \phi_i(u), \tag{12}$$

4.2 FUZZY MODEL PREDICTIVE CONTROLL STRUCTURE

Bringing together all the introduced methods, the final closed loop structure of the FMPC control system for the original nonlinear system can be illustrated as shown in figure (2).

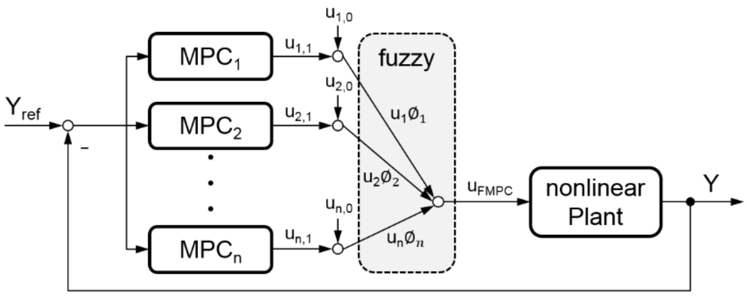

Figure 2: Workflow of a system with n MPC's and the fuzzyfication

In this case, the number of submodels is two and therefore two MPC's are merged according to the membership functions. The results of the convoluted network are discussed in the next section.

5. RESULTS

To demonstrate the results of the FMPC, a setpoint change from 903 K to 1073 K is investigated, since ϑ_g was the chosen partitioning variable to represent the overall plant behaviour. These setpoints correspond to the reference changes made in (Placek, 2015), where the demand temperature of the heated water was increased by $\Delta\vartheta = 4°C$. An extended Kalman-Filter was utilized for the nonlinear model state estimation. White noise was added for the simulation and the sampling time was set to 10 s for the discrete models.

The results of the FMPC and the two underlying MPC's are illustrated in figure (3) for the hot flue gas temperature and in figure (4) for the exhaust gas temperature. The time constant of the heat exchanger is very high and the temperature varies only slowly. The behaviour of all MPC's is therefore nearly the same and no proper distinction can be made, which is why this illustration is omitted. In figures (3) and (4), MPC$_1$ is designated as the one located at gap position 5, which corresponds to a centerpoint temperature of 733 K and MPC$_2$ is located at gap position 19 with a nominal flue gas temperature of 1173 K. For t = 0 min the composition of the FMPC is 5% MPC$_1$ and 95% MPC$_2$. After the reference step at t = 5 min, their membership changes to 15% for MPC$_1$ and to 85% for MPC$_2$. For the hot flue gas temperature the FMPC shows the lowest overshot but the highest rise time and all three MPC's have almost the same settling time. For the exhaust gas, the FMPC and MPC$_2$ do not really overshoot the reference value, whereas MPC_1 on the other hand does a little more, since it is further away from its original linearization point. The rise time for MPC_2 and the FMPC are also the same, whereas all three MPC's again have almost the same settling time.

What can be observed so far is, that some time before and after the reference step, all three MPC's seem to perform nearly equally for both setpoints. This could mean, that for the investigated model one MPC alone may would have been sufficient, since the encountered nonlinearities are not very strong. To show what advantages the implementation of fuzzy MPC's can have, an additional reference value was set above the aforementioned output constraints of 1273 K to 1380 K. This setpoint is far away from the perspective of MPC_1 and given the still active input constraints, not reachable any more. MPC_2 on the other hand is closer and can reach the desired reference value. The FMPC now has almost a 100% membership of MPC_2, due to the big distance to MPC_1. This example visualizes, how the implementation of a FMPC can ensure a consistent operation of the plant, although some local controller may fail to reach the desired setpoint or possibly not be stable any more at all.

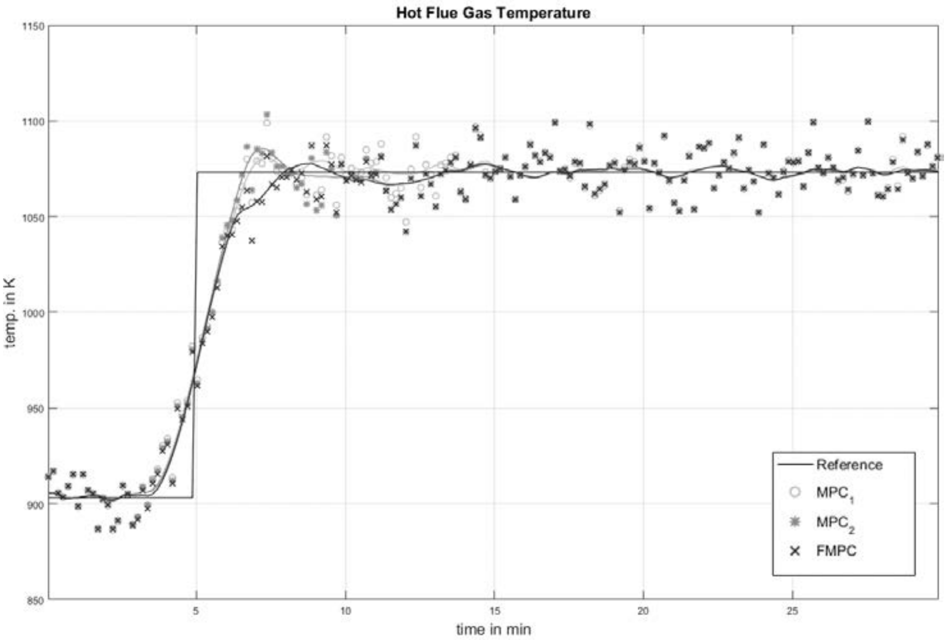

Figure 3: Simulation results of the FMPC for the hot flue gas

6. CONCLUSION

The implementation of a fuzzy model predictive controller for nonlinear biomass combustion plant models has been shown and that a good overall performance can be achieved, if at last one MPC is able to control the plant. The presented framework has many tuning possibilities starting at the modelling process and the MPC design up to the fuzzy-logic, which makes this approach very adaptable to different environments. A next step would be to implement the FMPC to a real plant and compare the results to other control algorithms.

LITERATURE

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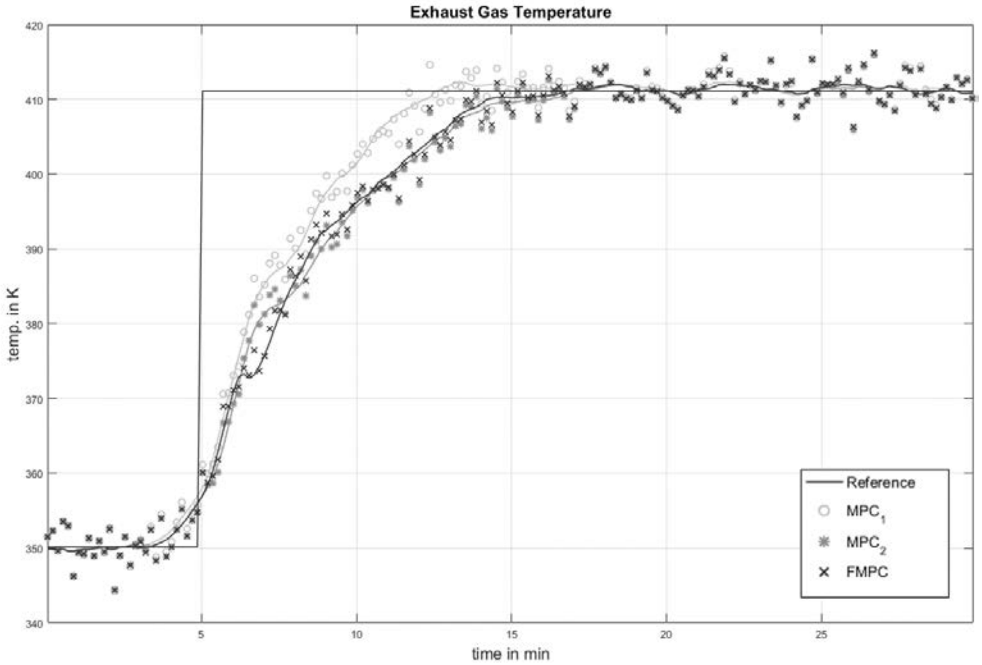

Figure 4: Simulation results of the FMPC for the exhaust gas temperature

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