Chapter 5
Multiple Support Random Vibrations of Beam Structures

Rudolf Heuer

Abstract The seismic behavior of elastic multi-span beams subjected to multiple support excitations is studied by means of a random vibration approach. Based on the common set of equations of motion an efficient formulation is developed in order to reduce the degrees of freedom. The resulting equations are formally identical to those that are valid for structures under uniform support excitations. Stationary random multiple support excitation is entirely performed by an approximate Pseudo Excitation Method calculating the power spectral density matrix of the structural response vector.

Keywords Multiple support excitation · Random vibrations · Multi-span beams

5.1 Introduction

Structures supported on several foundations such as bridges behave very complex when subjected to ground motions, e.g. earthquakes. Analysis of seismic response cannot be based on the single assumption that free-field ground motions are spatially uniform. Therefore, common discretization procedures, originally derived for structures under uniform support excitations, must be extended accordingly resulting in a larger system of equations of motion, see e.g. [1, 2].

The dynamic response of bridges subjected to deterministic multiple support excitation has been investigated by various researchers, [3, 4, 5]. Random vibrations of bridges have been analyzed generally by spectral analysis approach in the last two decades. In [6] the response of continuous two- and three-span beams to varying ground motions is evaluated and the validity of the commonly used assumption of equal support motion is examined. An extensive comparison of random vibration methods for multiple support seismic excitation analysis of long-span bridges can be found in [7]. Perotti [8] examines the structural response to non-stationary

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multiple-support random excitation in the frequency domain by means of evolutionary stochastic functions and parameters.

Allam et al. [9] treat cable-stayed bridges under multi-component random ground motion in frequency domain. In [10] the stochastic analysis of long span structures focuses on the site-response effect. The paper of Zanardo et al. [11] carries out a parametrical study of the pounding phenomenon associated with the seismic response of multi-span bridges with base isolation devices. Further comprehensive studies about spatial variation of seismic ground motions and its engineering application can be found in [12, 13].

In this contribution an advanced formulation for linear elastic multi-span beams under multiple support excitation is proposed in order to reduce the degrees of freedom in a mechanically consistent manner. The resulting differential equations are formally identical to those of structures under uniform support excitations. Applying the classical modal analysis approach, it becomes necessary to introduce time-dependent participation factors.

For stationary random multiple support excitation the Pseudo Excitation Method [14] is introduced, which includes the main effects of wave passage and site response.

This paper is related to applied structural dynamics. The treatment of the mechanical modeling of the multi-span beam under consideration is based on the formulation given in a previous paper of the author [15], where deterministic multiple support excitations are studied. In the present contribution it is intended to bring practical engineers closer to applied stochastic methods. Thus, on purpose, a strategy for treating the rather complex problem of multiple random support excitation has been generated that is quite easy to handle.

5.2 Governing Equations of Motion

The equation of motion of a discretized linear elastic beam subjected to *uniform* support excitation, (Fig. 5.1)

$$w_{g1}(t) = w_{g1}(t) = \cdots = w_{gM}(t) = w_g(t), \tag{5.1}$$

reads, compare [1],

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{c}\,\dot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = -\mathbf{m}\,\mathbf{e}^s\,\ddot{w}_g \tag{5.2}$$

where $\mathbf{m}, \mathbf{c}, \mathbf{k}$ stand for the mass, damping, and stiffniss matrix, respectively. $\mathbf{u}(t)$ denotes the vector of the nodal transverse deflections $w_i(t), i = 1, \ldots, N$. If the dicretization is extended to include also nodal rotations, $\mathbf{u}(t)$ contains additional rotatory degrees of freedom, and the corresponding system matrices have to be extended accordingly. The uniform ground acceleration is expressed by $\ddot{w}_g(t)$. The influence vector $\mathbf{e}^s$ represents the displacements (and rotations) of the masses resulting from

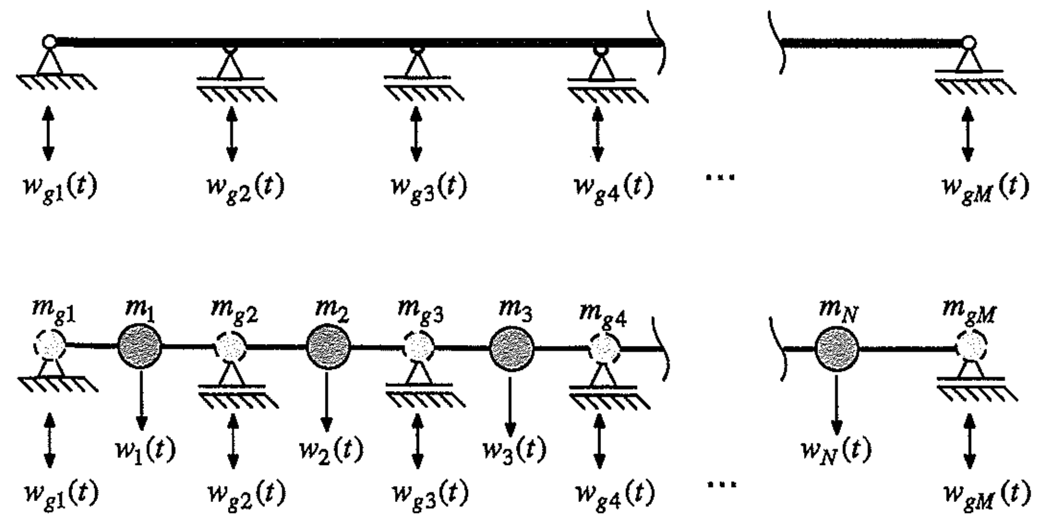

Fig. 5.1 Multi-span beam and its discretization as lumped mass model

static application of a ground displacement. In case of a lumped-mass model, where only nodal deflections (and no rotations) are considered, it is a vector with each element equal to unity, $(\mathbf{e}^s)^T = \begin{pmatrix} 1 \; 1 \; \dots \; 1 \end{pmatrix}$.

Contrary, the coupled equations of motion of multi-span beams under *multiple* support excitation can be written formally as, compare [2],

$$\begin{bmatrix} \mathbf{m} & \mathbf{m_g} \\ \mathbf{m_g}^T & \mathbf{m_{gg}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}^t \\ \ddot{\mathbf{u}}_g \end{bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c_g} \\ \mathbf{c_g}^T & \mathbf{c_{gg}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^t \\ \dot{\mathbf{u}}_g \end{bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k_g} \\ \mathbf{k_g}^T & \mathbf{k_{gg}} \end{bmatrix} \begin{bmatrix} \mathbf{u}^t \\ \mathbf{u}_g \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{p_g} \end{bmatrix}. \tag{5.3}$$

The displacement vector now contains two parts:

(a) $\mathbf{u}^t(t)$ includes the degrees of freedom of the beam, and
(b) $\mathbf{u_g}(t)$ contains the components of support excitation.

$\mathbf{m_g}, \mathbf{m_{gg}}, \mathbf{c_g}, \mathbf{c_{gg}}$, and $\mathbf{k_g}, \mathbf{k_{gg}}$ are submatrices associated with the support motion, and $\mathbf{p_g}(t)$ is the vector of support forces.

In the following a new, efficient representation of Eq. (5.3) is derived, which is related to the form of Eq. (5.2). Thus, it becomes possible to use numerical procedures that are common in the field of structures under uniform support excitation.

5.3 Modeling Procedure

In a first step the individually prescribed support displacements, $w_{gj}(t), j = 1, \dots, M$, are interpreted as additional degrees of freedom, i.e., $u_k(t), k = (N + 1), \dots, (N + M)$, see Fig. 5.2.

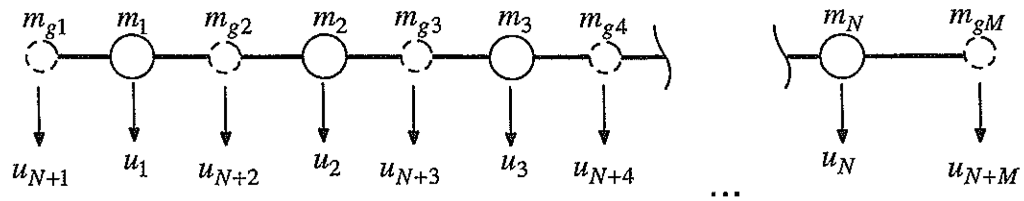

Fig. 5.2 Free body diagram of the lumped mass model

Next, the (singular) stiffness matrix of the complete discretized beam has to be evaluated, e.g., using the direct stiffness method by applying static unit deformations, which leads to

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12}\ldots & k_{1N} & k_{1(N+1)} & \ldots & k_{1(N+M)} \\ k_{21} & k_{22}\ldots & k_{2N} & \vdots & & \vdots \\ \vdots & \ddots & & & & \\ k_{N1} & \ldots & k_{NN} & k_{N(N+1)} & \ldots & k_{N(N+M)} \\ k_{(N+1)1} & \ldots & k_{(N+1)N} & k_{(N+1)(N+1)} & \ldots & k_{(N+1)(N+M)} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ k_{(N+M)1} & \ldots & k_{(N+M)N} & k_{(N+M)(N+1)} & \ldots & k_{(N+M)(N+M)} \end{bmatrix}. \tag{5.4}$$

Mass and damping matrices of Eq. (5.3) are of analogous form.

In the analysis of such dynamic system it is common to decompose the response into pseudo-static and dynamic components,

$$\mathbf{U}(t) = \begin{bmatrix} \mathbf{u}^{\mathrm{t}}(t) \\ \mathbf{u}_{\mathrm{g}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{u}^{\mathrm{s}}(t) \\ \mathbf{u}_{\mathrm{g}}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{u}(t) \\ \mathbf{0} \end{bmatrix}. \tag{5.5}$$

The pseudo-static component satisfies the equation

$$\begin{bmatrix} \mathbf{k} & \mathbf{k}_{\mathrm{g}} \\ \mathbf{k}_{\mathrm{g}}^{T} & \mathbf{k}_{\mathrm{gg}} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{\mathrm{s}}(t) \\ \mathbf{u}_{\mathrm{g}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_{\mathrm{g}}^{\mathrm{s}}(t) \end{bmatrix}, \tag{5.6}$$

from which one can solve for $\mathbf{u}^{\mathrm{s}}(t)$:

$$\mathbf{u}^{\mathrm{s}}(t) = -\mathbf{k}^{-1}\mathbf{k}_{\mathrm{g}}\mathbf{u}_{\mathrm{g}}. \tag{5.7}$$

Substituting Eqs. (5.5) and (5.7) into Eq. (5.3) results in

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{c}\,\dot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = -\left[\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_{\mathrm{g}}) + \mathbf{m}_{\mathrm{g}}\right]\ddot{\mathbf{u}}_{\mathrm{g}} - \left[\mathbf{c}\,(-\mathbf{k}^{-1}\mathbf{k}_{\mathrm{g}}) + \mathbf{c}_{\mathrm{g}}\right]\dot{\mathbf{u}}_{\mathrm{g}} \equiv \mathbf{p}_{eff}. \tag{5.8}$$

The vector of support forces can be expressed as

$$\mathbf{p}_\mathrm{g}(t) = (-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g}\mathbf{m}_\mathrm{g}^T + \mathbf{m}_\mathrm{gg})\ddot{\mathbf{u}}_\mathrm{g} + (-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g}\mathbf{c}_\mathrm{g}^\mathrm{T} + \mathbf{c}_\mathrm{gg})\dot{\mathbf{u}}_\mathrm{g} \\ + (-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g}\mathbf{k}_\mathrm{g}^T + \mathbf{k}_\mathrm{gg})\mathbf{u}_\mathrm{g} + \mathbf{m}_\mathrm{g}^T\ddot{\mathbf{u}} + \mathbf{c}_\mathrm{g}^\mathrm{T}\dot{\mathbf{u}} + \mathbf{k}_\mathrm{g}^T\mathbf{u}. \qquad (5.9)$$

The damping terms in the effective forcing function $\mathbf{p}_\mathrm{eff}$ can be neglected either the damping matrices are proportional to the stiffness matrices, e.i.,

$$\mathbf{c} = a_1\mathbf{k},\ \mathbf{c}_\mathrm{g} = a_1\mathbf{k}_\mathrm{g}, \qquad (5.10)$$

or the damping forces are assumed to be proportional to the relative velocity vector instead to the absolute velocity, i.e.,

$$\begin{bmatrix} \mathbf{c} & \mathbf{c}_\mathrm{g} \\ \mathbf{c}_\mathrm{g}^T & \mathbf{c}_\mathrm{gg} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^\mathrm{t} \\ \dot{\mathbf{u}}_\mathrm{g} \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{c} & \mathbf{c}_\mathrm{g} \\ \mathbf{c}_\mathrm{g}^T & \mathbf{c}_\mathrm{gg} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \mathbf{0} \end{bmatrix}, \qquad (5.11)$$

then Eq. (5.8) simplifies to

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\underbrace{\left[\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g}) + \mathbf{m}_\mathrm{g}\right]}_{\mathbf{M}}\ddot{\mathbf{u}}_\mathrm{g} \qquad (5.12)$$

This approximation is very common in structural dynamics, compare e.g. [2] or [13], and it can be assumed when the damping mechanism is mainly of the structural type.

Note that in case of a lumped-mass model, $\mathbf{m}_\mathrm{g}$ is a null matrix, which is assumed in all subsequent derivations.

Defining a non-dimensional ground acceleration vector,

$$\mathbf{F}_\mathrm{g}^\mathrm{T}(t) = \left[\ddot{u}_{g1}/\ddot{u}_{gref}\ \ \ddot{u}_{g2}/\ddot{u}_{gref}\ \cdots\ 1\ \ddot{u}_{gM}/\ddot{u}_{gref}\right], \qquad (5.13)$$

where $\ddot{u}_{gref} \neq 0$ represents a reference acceleration component, leads to

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g})\ddot{\mathbf{u}}_\mathrm{g} = -\mathbf{m}\mathbf{E}(t)\ddot{u}_{gref}, \qquad (5.14)$$

with the time-dependent influence vector

$$\mathbf{E}(t) = (-\mathbf{k}^{-1}\mathbf{k}_\mathrm{g})\mathbf{F}_\mathrm{g}(t). \qquad (5.15)$$

When comparing Eqs. (5.14) to (5.2) of the beam under uniform support excitation it turns out that both are of the same dimension and structure.

An application of that efficient formulation to deterministic multiple support excitation is given in [15]. There, applying the classical modal analysis approach,

$$\mathbf{u}(t) = \sum_{i=1}^{N} \vec{\phi}_i y_i(t), \tag{5.16}$$

where $\vec{\phi}_i$ represents the eigenvectors, and y_i stands for the generalized coordinates, the resulting uncoupled equations of motion are of the form

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\Gamma_i(t)\ddot{u}_{gref}, \tag{5.17}$$

with time-dependent participation factors

$$\Gamma_i(t) = \frac{\vec{\phi}_i^T \mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_g)}{\vec{\phi}_i^T \mathbf{m}\vec{\phi}_i} \mathbf{F}_g^T(t). \tag{5.18}$$

5.4 Stationary Random Excitation

5.4.1 An Approximate Excitation Model

Under close examination of the seismic analysis of multiply supported bridge structures subjected to spatially varying ground motion three main effects have to be taken into account compare [16]:

(a) Wave passage, considering the difference in the arrival times of the waves at stations located apart due to the finite nature of the seismic wave velocities,
(b) Incoherence, caused due to wave propagation in a heterogeneous medium with numerous reflections and refractions,
(c) Site response, considering local soil conditions.

In the previous work [15] about deterministic excitation only the wave passage effect has been considered.

The present paper introduces an approximate procedure for random vibrations, the Pseudo Excitation Method (PEM), see e.g. [14], which includes both the cross-correlation terms between the participant modes and between the excitations.

Local effects are treated by assuming different power spectral densities (PSDs) of the ground acceleration at each support,

$$S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega) = \lambda_k S_{\ddot{u}_{g1}\ddot{u}_{g1}}(\omega) = \lambda_k S_a(\omega), \tag{5.19}$$

where it is suggested that the factor λ_k can be estimated by the ratio of individual mean square values,

$$\lambda_k = \frac{\int_0^{\omega_1} S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega)d\omega}{\int_0^{\omega_1} S_{\ddot{u}_{g1}\ddot{u}_{g1}}(\omega)d\omega} = \frac{\int_0^{\omega_1} S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega)d\omega}{\int_0^{\omega_1} S_a(\omega)d\omega}. \tag{5.20}$$

As example, the common approach of the filtered Kanai-Tajimi spectral density could be used, see e.g. [1]:

$$S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega) = S_0 \left| H_g(\omega) \right|^2 \left| H_f(\omega) \right|^2, \tag{5.21}$$

where

$$\left| H_g(\omega) \right|^2 = \frac{\left[1 + 4\zeta_{gk}^2 \left(\omega/\omega_{gk} \right)^2 \right]}{\left[1 - \left(\omega/\omega_{gk} \right)^2 \right]^2 + 4\zeta_{gk}^2 \left(\omega/\omega_{gk} \right)^2} \tag{5.22}$$

is the squared Kanai-Tajimi frequency response function and

$$\left| H_f(\omega) \right|^2 = \frac{\left(\omega/\omega_{fk} \right)^4}{\left[1 - \left(\omega/\omega_{fk} \right)^2 \right]^2 + 4\zeta_{fk}^2 \left(\omega/\omega_{fk} \right)^2} \tag{5.23}$$

denotes the high pass filter frequency response function.

In the next step, this random excitation is replaced by a pseudo sinusoidal excitation, where the first ground node is taken as reference node,

$$\ddot{u}_{gref}(t) = \ddot{u}_{g1}(t) = \sqrt{S_a(\omega)} \exp(i\omega t). \tag{5.24}$$

The time delay of the ground motion depends on the distance of ground nodes j measured to the reference node 1,

$$T_j = \left| u_{gj} - u_{g1} \right| / v_{app}, \tag{5.25}$$

where v_{app} denotes the surface apparent wave velocity. Finally, the vector of pseudo sinusoidal excitation becomes

$$\ddot{\mathbf{u}}_g = \ddot{\mathbf{U}}_g \exp(i\omega t) = \mathbf{d}(i\omega) \sqrt{S_a(\omega)} \exp(i\omega t) = \mathbf{d}(i\omega)\, \ddot{u}_{gref}(t), \tag{5.26}$$

with the non-dimensional complex vector

$$\mathbf{d}(i\omega) = \left[1 \ \sqrt{\lambda_2} \exp(-i\omega T_2) \ \cdots \ \sqrt{\lambda_M} \exp(-i\omega T_M) \right]^T. \tag{5.27}$$

5.4.2 *Computation of Structural Response*

The use of PEM makes it possible to determine the PSDs of the dynamic response.

Thereby the vector of the total response is formulated by means of a time-harmonic Ansatz,

$$\tilde{\mathbf{u}}^{t}(t) = \tilde{\mathbf{u}}(t) + \tilde{\mathbf{u}}^{s}(t) = \left[\tilde{\mathbf{U}}(i\omega) + \tilde{\mathbf{U}}^{s}(i\omega)\right]\exp(i\omega t). \tag{5.28}$$

Solving the equation of motion associated to $\tilde{\mathbf{u}}(t)$, compare Eq. (5.14),

$$\mathbf{m}\,\ddot{\tilde{\mathbf{u}}} + \mathbf{c}\,\dot{\tilde{\mathbf{u}}} + \mathbf{k}\,\tilde{\mathbf{u}} = -\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_g)\ddot{\tilde{\mathbf{u}}}_g = -\mathbf{m}\,\mathbf{E}\,\ddot{u}_{gref}, \tag{5.29}$$

with the complex influence vector

$$\mathbf{E}(i\omega) = (-\mathbf{k}^{-1}\mathbf{k}_g)\mathbf{d}(i\omega), \tag{5.30}$$

gives the complex amplitude vector of the dynamic part of nodal displacements

$$\tilde{\mathbf{U}}(i\omega) = -\mathbf{H}(i\omega)\mathbf{m}\mathbf{E}(i\omega)\sqrt{S_a(\omega)}, \tag{5.31}$$

where the complex transfer matrix is defined as

$$\mathbf{H}(i\omega) = \left[\mathbf{k} + i\omega\mathbf{c} - \omega^2\mathbf{m}\right]^{-1}. \tag{5.32}$$

The pseudo-static contribution, see Eqs. (5.7) and (5.26), becomes

$$\tilde{\mathbf{U}}^{s}(i\omega) = -\mathbf{k}^{-1}\mathbf{k}_g\,\tilde{\mathbf{U}}_g = \frac{1}{\omega^2}\mathbf{k}^{-1}\mathbf{k}_g\,\ddot{\tilde{\mathbf{U}}}_g = \frac{\sqrt{S_a(\omega)}}{\omega^2}\,\mathbf{k}^{-1}\mathbf{k}_g\,\mathbf{d}. \tag{5.33}$$

Finally the total pseudo structural displacement vector reads

$$\tilde{\mathbf{u}}^{t}(t) = \tilde{\mathbf{U}}^{t}(i\omega)\exp(i\omega t), \quad \tilde{\mathbf{U}}^{t}(i\omega) = \left[\frac{1}{\omega^2}\mathbf{k}^{-1}\mathbf{k}_g\mathbf{d} - \mathbf{H}(i\omega)\mathbf{m}\mathbf{E}\right]\sqrt{S_a(\omega)}, \tag{5.34}$$

and the corresponding matrix of PSDs can be expressed as

$$[\mathbf{S}_{\tilde{\mathbf{u}}^t\tilde{\mathbf{u}}^t}(\omega)] = \left[\tilde{\mathbf{U}}^{t*}\right]\left[\tilde{\mathbf{U}}^{t}\right]^{T}, \tag{5.35}$$

where the superscript * represents the complex conjugate of the vector.

5.5 Conclusions

A new formulation for linear elastic multi-span beams under multiple support excitation has been proposed in order to reduce the degrees of freedom in a mechanically consistent manner. The resulting differential equations are formally identical to those of structures under uniform support excitations. Thus, in case of deterministic excitation it becomes possible to apply only slightly modified procedures for treating vibrations of structures under uniform support excitation. Making use of modal analysis, e.g., it becomes necessary to introduce time-dependent participation factors. For stationary random multiple support excitation an approximate procedure, the Pseudo Excitation Method is introduced, which includes the main effects of wave passage and site response.

The mechanical modeling of the considered structural problem and its combination with random vibrations are new and valuable in applied engineering.

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