Reduction of the 6th and 12th harmonic in the torque ripple of a salient pole synchronous reluctance machine

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Abstract—In this paper a reduction method for torque-ripple is evaluated. Synchronous reluctance motors (SynRM) combine highly efficient operation with low initial costs of the drive system. The disadvantage of the used salient pole rotor structure is that a strong torque-ripple could occur. To deal with this unwanted torque harmonics the classical mathematical model of the SynRM has to be enhanced. With this extended model a torque-ripple suppression method was developed which does not require detailed information of the system. These theoretical approaches were experimentally tested on a prototype to verify the performance of the enhanced drive system.

Keywords—synchronous reluctance machine, torqueripple, harmonics, real-time compensation.

I. INTRODUCTION

Synchronous–reluctance machines (SynRM) are getting more popular in the last years. This is caused by the higher efficiency compared to widely used induction motors (IM) [1][2][3][4] and the lower initial costs compared to the permanent magnet synchronous motors (PMSM). Furthermore the use of a position-sensorless method, which replaces the angular sensor by mathematical equations [5][6][7], is possible. This also increases the economical benefits by decreasing failure probability. The disadvantage of SynRMs, especially in the salient pole configuration (see Fig. 1), is the increased generation of unwanted harmonics in torque [8].



Fig. 1. Position of the dq-reference frame in the used rotor geometry

The key data of the used SynRM–prototype are listed in Tab. I, where the rotor lamination cross section is shown in Fig. 1 [8].

	Maximum Shaft Power	$185\mathrm{W}$		
	Maximum Shaft Torque	$0.6~\mathrm{N}~\mathrm{m}$		
	Nominal Shaft Speed	$3000\mathrm{rpm}$		
	Line Current	1.5 A RMS		
	Line Voltage	162 V RMS		
ABLE I.	KEY DATA OF THE USED SYNRM-PROTOTYPE WITH			
	SALIENT PO	LES		

II. TORQUE-RIPPLE

By variation of the used rotor structure it is possible to influence the characteristics of the SynRM. For example the choice of a salient pole rotor can increase the efficiency of the motor compared to the flux barrier rotor type. But this salient pole configuration increases also the occurrence of disturbing harmonics in the drive system, which is a big disadvantage [8].

To characterise these unwanted harmonics (in our case the torque–ripple) the SynRM was driven in the common used field-oriented control operation mode. The torque t was determined through equation (5). Of course the torque determination is also possible by a torque sensor, but this reduces dramatically the dynamic of the system caused by the limiting sensor bandwidth. Fig. 2 shows the measured spectrum of the determined torque at various speeds ω_m and as a function of the stator-current space-vectors magnitude $|\underline{i}_{\alpha,\beta}|$. The two most dominant harmonics in this machine are the 12th followed by the 6th. In the following the focus of this paper lies on these two harmonic components.

III. MATHEMATICAL DESCRIPTION OF SYNCHRONOUS-RELUCTANCE MOTORS

The most comfortable way to describe a SynRM is in the rotor-fixed reference frame, the dq-reference frame. The d-axis is aligned in the direction of maximum magnetic permeance, where the q-axis is aligned in the direction of maximum magnetic reluctance. Figure 1 shows the positions of these axis in the used rotor configuration. The difference between the position of the d-axis and the symmetry axis of the rotor-pole arises from the asymmetrical rotor structure ($\beta_P \neq 90^\circ$).

A. Fundamental Wave Model

In this section the common fundamental wave model of the SynRM is presented. To describe the rated magnetic



Fig. 2. Measured spectrum of the torque (uncompensated system) as a function of the speed ω_m and the magnitude of the stator-current space-vector $|i_{\alpha,\beta}|$

flux linkage of the motor equation (1) and (2) are used.

$$\psi_d = l_d(i_d) \, i_d \tag{1}$$

$$\psi_q = l_q(i_q) \, i_q \approx l_q \, i_q \tag{2}$$

In the case of a SynRM with salient poles the crosscoupling between the direct-axis and the quadrature-axis is negligibly small and can be ignored. So the equations only consist of the rated direct-inductance $l_d(i_d)$, the quadrature-inductance l_q , the direct-current i_d and the quadrature-current i_q . Figure 3 shows the measured inductance-characteristics $l_d(i_d)$ and $l_q(i_q) \approx l_q$ of the used SynRM.



Fig. 3. Measured rated inductances $l_d(i_d)$ and $l_q(i_q)$

With these specific equations of this SynRM type the fundamental wave model of the stator voltage equations can be written in the usual way like shown in (3) and (4).

$$u_d = r \, i_d + l_d(i_d) \, \frac{di_d}{d\tau} - \omega_m \, l_q \, i_q \tag{3}$$

$$u_q = r i_q + l_q \frac{di_q}{d\tau} + \omega_m \, l_d \, i_d \tag{4}$$

Voltages u_d and u_q are the components of the rated stator-voltage space-vector, r represents the stator-resistance and ω_m is the motor speed.

Finally the equation of the rated torque t is given by (5).

$$t = -Im(\psi \,\underline{i^*}) = (l_d(i_d) - l_q) \, i_d \, i_q \tag{5}$$

B. Extended Mathematical Model

To handle the unwanted torque-ripple (see section II) the introduced fundamental model of section III-A is unsufficient. An extension of this model is necessary to describe the disturbing harmonics. An introduction of position-angle dependent inductances $l_d(\gamma)$ and $l_q(\gamma)$ is recommended. Assuming constant speed ω_m allows the transformation from a position-angle dependency to a time dependency of the inductances given by (6) and (7).

$$l_d(\tau) = l_{d,0} + \sum_{k=2}^{\infty} \left(l_{d,k}(\tau) \right) = l_{d,0} + l_{d,\Sigma}$$
(6)

$$l_q(\tau) = l_{q,0} + \sum_{k=2}^{\infty} (l_{q,k}(\tau)) = l_{q,0} + l_{q,\Sigma}$$
(7)

The variables $l_{d,0}$ and $l_{q,0}$ represent the time independent part of the inductances like described in Sec.

III-A. $l_{d,k}(\tau)$ and $l_{q,k}(\tau)$ are used to describe the effects of the k^{th} harmonic. To keep the mathematical model of the inductance saturation as simple as possible the inductance $l_d(i_d(\tau), \tau)$ is assumed to be independent of the current ripple component $i_{d,k}(\tau)$, which means $l_d(i_d(\tau), \tau) \approx l_d(i_{d,0}, \tau) = l_d(\tau)$.

Consequently, these time dependent components in the inductances lead to time dependent components in the current and in the voltage like mentioned in (8) - (11).

$$u_d(\tau) = u_{d,0} + \sum_{k=2}^{\infty} \left(u_{d,k}(\tau) \right) = u_{d,0} + u_{d,\Sigma}$$
 (8)

$$u_q(\tau) = u_{q,0} + \sum_{k=2}^{\infty} \left(u_{q,k}(\tau) \right) = u_{q,0} + u_{q,\Sigma}$$
(9)

$$i_d(\tau) = i_{d,0} + \sum_{k=2}^{\infty} (i_{d,k}(\tau)) = i_{d,0} + i_{d,\Sigma}$$
 (10)

$$i_q(\tau) = i_{q,0} + \sum_{k=2}^{\infty} (i_{q,k}(\tau)) = i_{q,0} + i_{q,\Sigma}$$
 (11)

Considering these time dependent inductances $l_d(\tau)$ and $l_q(\tau)$ in the stator–voltage equations (3) and (4) lead to modified stator–voltage equations (12) and (13).

$$u_d = r i_d + \frac{dl_d}{d\tau} i_d + l_d \frac{di_d}{d\tau} - \omega_m l_q i_q \qquad (12)$$

$$u_q = r i_q + \frac{dl_q}{d\tau} i_q + l_q \frac{di_q}{d\tau} + \omega_m l_d i_d \qquad (13)$$

Inserting (6) - (11) to this extended model results in the following form of the stator–voltage equations (14) and (15).

$$u_{d,0} + u_{d,\Sigma} = r (i_{d,0} + i_{d,\Sigma}) + \frac{d l_{d,\Sigma}}{d \tau} (i_{d,0} + i_{d,\Sigma}) + (l_{d,0} + l_{d,\Sigma}) \frac{d i_{d,\Sigma}}{d \tau} - \omega_m (l_{q,0} + l_{q,\Sigma}) (i_{q,0} + i_{q,\Sigma})$$
(14)

$$u_{q,0} + u_{q,\Sigma} = r (i_{q,0} + i_{q,\Sigma}) + \frac{d l_{q,\Sigma}}{d \tau} (i_{q,0} + i_{q,\Sigma}) + (l_{q,0} + l_{q,\Sigma}) \frac{d i_{q,\Sigma}}{d \tau} + \omega_m (l_{d,0} + l_{d,\Sigma}) (i_{d,0} + i_{d,\Sigma})$$
(15)

Now it is possible to separate the time independent terms (the fundamental wave terms) and transform it to the fundamental wave inductances (16) and (17).

$$l_{d,0} = \frac{u_{q,0} - r \, i_{q,0}}{\omega_m \, i_{d,0}} \tag{16}$$

$$l_{q,0} = -\frac{u_{d,0} - r \, i_{d,0}}{\omega_m \, i_{q,0}} \tag{17}$$

The remaining terms represent the harmonic part of the system. Linearising this non-linear system of differential equations in an operation point provides following equations (18) and (19).

$$0 \approx \sum_{k=2}^{\infty} \left(i_{d,0} \cdot \frac{d \, l_{d,k}}{d\tau} + \frac{d \, i_{d,k}}{d\tau} \cdot l_{d,0} - \omega_m \, i_{q,0} \cdot l_{q,k} \right)$$

$$+ r \, i_{d,k} - u_{d,k} - \omega_m \, l_{q,0} \, i_{q,k} \right)$$

$$0 \approx \sum_{k=2}^{\infty} \left(i_{q,0} \cdot \frac{d \, l_{q,k}}{d\tau} + \frac{d \, i_{q,k}}{d\tau} \cdot l_{q,0} + \omega_m \, i_{d,0} \cdot l_{d,k} \right)$$

$$+ r \, i_{q,k} - u_{q,k} + \omega_m \, l_{d,0} \, i_{d,k} \right)$$
(18)
(19)

After Laplace-transforming the linear equations (18) and (19) it is possible to resolve these equations to the inductances of each harmonic k in the Laplace-domain as shown in (20) and (21).

$$L_{d,k} = \frac{1}{i_{d,0}} \left[\frac{s}{s^2 + \omega_m^2} \left(\bigcup_{d,k} - r \, I_{d,k} \right) + \frac{\omega_m}{s^2 + \omega_m^2} \left(\bigcup_{d,k} - r \, I_{d,k} \right) \right]$$

$$L_{q,k} = \frac{1}{i_{q,0}} \left[-\frac{\omega_m}{s^2 + \omega_m^2} \left(\bigcup_{d,k} - r \, I_{d,k} \right) + \frac{s}{s^2 + \omega_m^2} \left(\bigcup_{d,k} + r \, I_{d,k} \right) - l_{q,0} \, I_{q,k} \right]$$
(20)
(21)

The used variables are defined as follows

$$l_{d,k} \circ \underbrace{\bullet}_{\tilde{d}d,k} \qquad \qquad l_{q,k} \circ \underbrace{\bullet}_{\tilde{d}q,k} \qquad (22)$$

$$u_{d,k} \circ - \bullet U_{d,k} \qquad u_{q,k} \circ - \bullet U_{a,k}$$
(23)

$$i_{d,k} \circ \underbrace{\bullet}_{\widetilde{d},k} f_{d,k} = i_{q,k} \circ \underbrace{\bullet}_{\widetilde{d},k} f_{q,k}.$$
 (24)



Fig. 4. Measurement of the currents i_d and i_q and the voltages u_d and u_q in a conventional field-oriented control

To consider the voltages and currents appropriately, the measurements shown in Fig. 4 are used. In this special case the focus is on the most dominant 12th harmonic component of the spectrum. Out of the measurement it is possible to characterise the required currents and voltages as

$$i_d = i_{d,0} + \hat{i}_{d,12} \cos(12\,\omega_m\,\tau)$$
 (25)

$$i_q = i_{q,0} + i_{q,12} \cos(12\,\omega_m\,\tau)$$
 (26)

$$u_d = u_{d,0} - \hat{u}_{d,12} \,\cos(12\,\omega_m\,\tau) \tag{27}$$

$$u_q = u_{q,0} - \hat{u}_{q,12} \cos(12\,\omega_m\,\tau).$$
 (28)

Together with the inductances in the Laplace-domain (20) and (21) the 12^{th} harmonic of the inductances in the time-domain can be given to (29) and (30).

$$l_{d,12} \approx \frac{\hat{u}_{d,12} + r\,\hat{i}_{d,12}}{12\,\omega_m\,\frac{1}{i_{d,0}}} \cdot \sin(12\,\omega_m\tau)$$

$$+ l_{d,0}\,\frac{\hat{i}_{d,12}}{i_{d,0}} \cdot \cos(12\,\omega_m\,\tau)$$

$$l_{q,12} \approx \frac{\hat{u}_{q,12} + r\,\hat{i}_{q,12}}{12\,\omega_m\,\frac{1}{i_{q,0}}} \cdot \sin(12\,\omega_m\tau)$$

$$+ l_{q,0}\,\frac{\hat{i}_{q,12}}{i_{q,0}} \cdot \cos(12\,\omega_m\,\tau)$$
(30)

Applying this approach in an analogous way to the next dominant 6^{th} harmonic of the motor provides the 6^{th} harmonic component of the direct–inductance $l_{d,6}$ and the quadrature–inductance $l_{q,6}$.

With these three components of l_d and l_q the mathematical model can be extended to describe the two most dominant harmonic effects of this SynRM (see (31) and (32)).

$$l_d(\tau) = l_{d,0} + \sum_{k=2}^{\infty} \left(l_{d,k}(\tau) \right) \approx l_{d,0} + l_{d,6}(\tau) + l_{d,12}(\tau)$$
(31)

$$l_q(\tau) = l_{q,0} + \sum_{k=2}^{\infty} \left(l_{q,k}(\tau) \right) \approx l_{q,0} + l_{q,6}(\tau) + l_{q,12}(\tau)$$
(32)

Fig. 5 shows the FEM-simulation of the inductances $l_d(\tau)$ and $l_q(\tau)$. The 6th and the 12th harmonic can be easily seen in this graph, which verifies the extended model of the SynRM introduced above.

Now with this extended model of the SynRM with salient poles the design of an appropriate control system is possible.

IV. CONCEPT OF CONTROL SYSTEM

In the following section a method to reduce the torque–ripple is presented. First of all there have to be made a differentiation between the ripple generation caused by the PWM module of the drive system [9] and



Fig. 5. Simulation of the rated direct– and quadrature–inductances $l_d(\tau)$ and $l_q(\tau)$

the ripple generation caused by the motor itself. In this paper the second part is covered.

The easiest way to reduce the unwanted harmonics is to choose an appropriate rotor design of the SynRM, but this can lead to a reduction of the motor efficiency [8]. Another possibility, which is chosen in this paper, is a feed-forward control of the torque disturbances. The key-point of this method is the determination of the torque-ripple. One way to find the optimal feedforward signal is by the use of a fuzzy-logic or by an adaptive algorithm [10]. This entails in the loss of determinism, which could be an essential disadvantage. Another option to find an optimal reference signal is by a very detailed FEM-simulation of the motor or by empirical experiments. This solution can be stored in a lookup-table of the drive system to provide a efficient solution of this problem [11][12]. The main disadvantage of this solution is the strong parameter dependency of used motor, which significantly decreases the robustness and the adaptiveness of the control system.

In this paper a synchronous-demodulator (see Fig. 6) is used to estimate the ideal feed-forward signal for torque-ripple suppression [13].



Fig. 6. Synchronous-demodulator

The biggest advantage of this way of torque-ripple estimation is the fact that there is minimal systemknowledge necessary to suppress the disturbances. Only the rated speed ω_m and the order of the harmonic of the torque-ripple k has to be known, which allows a very robust behaviour of the control system with regard to parameter variations. The input for the synchronousdemodulator is the torque signal t. This signal could be estimated through equation (5) or could come from a torque sensor. Both variants were implemented and tested, but this work only describes the variant with the calculated torque, because of the higher dynamic of the system (no bandwidth limitation through torque sensor). ε and α are used as tuning parameters of the control system to guarantee the greatest possible robustness of the system.

The transfer-function $N_k(s)$ of the synchronousdemodulator is given in equation (33), where the dependency on the tuning parameter α is shown in the corresponding frequency response in Fig. 7. Out of the transfer-function $N_k(s)$ and the bode-plot the bandpass characteristic of the synchronous-demodulator is obvious.

 $N_{\widetilde{\omega}}(s) = \varepsilon \cdot \frac{s \, \cos(\alpha) - k \, \omega_m \, \sin(\alpha)}{s^2 + (k \, \omega_m)^2}$

(33)



Fig. 7. Frequency responses of the synchronous–demodulator depending on the tuning–parameter α

Now the whole control system with disturbance-feedforward can be specified. Fig. 8 shows in the lower part of the figure the common field-oriented control of a SynRM. $i_{d,ref}$ and $i_{q,ref}$ represent the reference-currents from the speed-control, where $i_{d,act}$ and $i_{q,act}$ are the actual measured currents of the motor. t represents the torque signal of the system, which is fed to both of the synchronous-demodulators N_6 and N_{12} to estimate the 6^{th} and the 12^{th} harmonic of the torque disturbances.

There is a separated current–control for each of the two currents in the d, q–reference frame. This current–control generates the wanted stator–voltage space–vector, which will be realised by the PWM module of the voltage–source inverter. To do the transformations from the α, β – and the d, q–reference frame and back, an angular–positions sensor is used. Of course it is supposable that a position–sensorless method could replace this sensor to decrease the initial system–costs.



Fig. 8. Control structure to reduce the 6^{th} and 12^{th} harmonic disturbances in torque t



Fig. 9. Measurement of the torque t without compensation algorithm (upper figure) and with compensation algorithm (lower figure)

To analyse the stability of this method the extended mathematical model, which is introduced in Sec. III-B, is used. The system was linearised in various operating– points where the stability–proof was made by pole–zero map analyse of the system. To optimise the robustness of the system the damping–factor ξ_i (see (34)) was maximised by variation of the tuning parameters ε and α . κ_i and μ_i describes the poles of the system according to $\lambda_i = \kappa_i + j \mu_i$.

$$\xi_i = \frac{-\kappa_i}{\sqrt{\kappa_i^2 + \mu_i^2}} \tag{34}$$

V. EXPERIMENTAL RESULTS

Finally the described control strategy (see Sec. IV) was implemented on the drive system of the SynRM prototype.

Fig. 9 demonstrates the functionality of the torqueripple reduction method. In the upper part of this figure



Fig. 10. Measured spectrum of the torque (compensated system) as a function of the speed ω_m and the magnitude of the stator-current space-vector $|i_{\alpha,\beta}|$

Speed	Order of harmonic	$ \underline{i}_{lpha,eta} $	Amplification torque-ripple	Amplification voltage-ripple
750 rpm 1500 rpm 2250 rpm 3000 rpm	6 6 6 6	1 1 1 1	$-26.7 \mathrm{dB}$ $-16.2 \mathrm{dB}$ $-34.6 \mathrm{dB}$ $-51.1 \mathrm{dB}$	0.1 dB 0.1 dB 6.4 dB 16.2 dB
$750 { m rpm}$ $1500 { m rpm}$ $2250 { m rpm}$ $3000 { m rpm}$	12 12 12 12 12	1 1 1 1	$-30.3 \mathrm{dB}$ $-34.6 \mathrm{dB}$ $-35.6 \mathrm{dB}$ $-52.9 \mathrm{dB}$	9.5 dB 19.5 dB 18.6 dB 17.8 dB

 TABLE II.
 CHARACTERISATION OF THE MEASURED TORQUE-RIPPLE REDUCTION

the uncompensated version of the drive system is shown, where in the lower part of the figure the ripple suppression method is active. The torque signal t becomes very smooth, where the stator-voltage u_{α} gets increased ripple components compared to the uncompensated case.

To verify this suppression method in the whole speed range, the spectrum of the compensated system was determined as shown in Fig. 10. The spectrum was measured at four different speeds ω_m and plotted as a function of the magnitude of the stator-current space-vector $|\underline{i}_{\alpha,\beta}|$. Compared to the spectrum of the uncompensated drive system (see Fig. 2) the peaks at the 6th and the 12th harmonic are significantly reduced. This fact is also specified in Tab. II. The 6th and the 12th torque–ripple component can be reduced significantly, where no other torque–ripple components were generated by the suppression–method.

VI. CONCLUSION

In this paper a very robust method of torque-ripple suppression was presented. An extended mathematical model of the SynRM was introduced, which allows the consideration of the harmonic effects of the salient pole SynRM. The presented torque–ripple reduction method does not need much information of the system, which allows universal operation of different SynRMs with minimal tuning effort on the control system. Experiments on the prototype verified the functionality of the control system and demonstrated the performance with respect to torque–ripple suppression.

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