Real-time closed-loop control for micro mirrors with quasistatic comb drives

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ABSTRACT

This paper presents the application of a real-time closed-loop control for the quasistatic axis of electrostatic micro scanning mirrors. In comparison to resonantly driven mirrors, the quasistatic comb drive allows arbitrary motion profiles with frequencies up to its eigenfrequency. A current mirror setup at Fraunhofer IPMS is manufactured with a staggered vertical comb (SVC) drive and equipped with an integrated piezo-resistive deflection sensor, which can potentially be used as position feedback sensor. The control design is accomplished based on a nonlinear mechatronic system model and the preliminary parameter characterization. In previous works [1,2] we have shown that jerk-limited trajectories, calculated offline, provide a suitable method for parametric trajectory design, taking into account physical limitations given by the electrostatic comb and thus decreasing the dynamic requirements. The open-loop control shows in general unfavorable residual eigenfrequency oscillations leading to considerable tracking errors for desired triangle trajectories [3]. With real-time closed-loop control, implemented on a dSPACE® system using an optical feedback, we can significantly reduce these errors and stabilize the mirror motion against external disturbances. In this paper we compare linear and different nonlinear closed-loop control strategies as well as two observer variants for state estimation. Finally, we evaluate the simulation and experimental results in terms of steady state accuracy and the concept feasibility for a low-cost realization.

Keywords: real-time closed-loop control, quasistatic microscanner, electrostatic staggered vertical comb drive, flatness-based and sliding-mode control, normal form and high-gain observer

1. INTRODUCTION

Micro scanning mirrors play an important role in various applications for highly miniaturized, reliable and cost efficient scanning systems like compact laser projection displays [4,5], portable scanning grating spectrometers [6] or LIDAR [7]. These MEMS devices feature a low cost, CMOS compatible production process as well as high accuracy and high scan rate properties. Compared to Lissajous figure scanning followed by resonant 2D scanning systems [4], quasistatic microscanners can perform arbitrary trajectories or tilt statically with high precision and repeatability. The presented cardanic 2D quasistatic resonant scanner, Fig. 1, can realize a raster scan by combining a sawtooth motion of the outer quasistatic axis and a resonant motion of the inner resonant axis. The quasistatic axis of the 2D micro scanner, cf. Fig. 1b,1d, tilts electrostatically driven using a staggered vertical comb drive. This very low damped spring mass system has to be controlled to realize high precise tracking applications.

In previous works, we concentrated on open-loop control strategies [1,3,8]. In this contribution we compare and validate the real-time closed-loop control with a robust linear PID controller [9] and with nonlinear closed-loop control methods: flatness-based control as system inversion and sliding-mode control. The sliding-mode
control [10–12] achieves exceptional tracking performance by high frequency switching. The resulting characteristic chattering poses no problem for the micro mirror electrostatic comb drive. An external optical sensor serves for measuring the micro mirror tilt position. As an alternative to this optical feedback considered in this paper, recent technology developments in piezo-sensors show the potential to serve as an internal measurement for closed loop feedback control [13].

After introducing the system modeling and state of the art open-loop control techniques in section 2, we derive the closed-loop control strategies in section 3 and demonstrate the experimental results in section 4.

(a) Photograph of 2D quasistatic-resonant microscanner chip [14]; the mirror size is 2.6 mm × 3.6 mm
(b) Detailed photograph of quasistatic comb drive showing the arrangement of the comb electrodes, spring and the piezo-resistive sensor [15, 16]

(c) Static voltage deflection characteristic $\theta(v_{1,2})$ up to max. deflection angle of $\pm10^\circ$ with $\tau_{1,\text{max}} = 145$ V for positive deflection $\theta > 0$ and $\tau_{2,\text{max}} = 143$ V for negative deflection $\theta < 0$
(d) Principal design of quasistatic staggered vertical comb drive showing electrical connections and torques [1]

Figure 1: 2D quasistatic-resonant microscanner chip from TACO project [14]

2. OPEN-LOOP CONTROL OF QUASISTATIC COMB DRIVE

The control aims at generating the input voltages for tracking a desired deflection trajectory. Therefore we create a physical model of the mechatronic system, cf. Fig. 1d, and identify its parameters as follows. We suppose the...
torque equilibrium Eq. (1), neglecting external disturbances:

\[
J\ddot{\theta} + b\dot{\theta} + \tau_s(\theta) = 1/\tau_1 + 1/\tau_2,
\]

where \(\theta, \dot{\theta}, \ddot{\theta}\) denote the deflection angle and its time derivatives, \(v_1\) and \(v_2\) are the applied driving voltages at the comb electrodes, \(\tau_1\) and \(\tau_2\) describe the electrostatic drive torques and the mirror parameters are: mirror inertia \(J\), linear viscous damping \(b\), nonlinear spring torque \(\tau_s(\theta)\), and the capacitance derivatives \(C_1 = 2/\tau_1\) and \(C_2 = 2/\tau_2\). We determined the mirror inertia \(J = 3.89 \times 10^{-12} \text{kgm}^2\) with an Ansys finite element model. By exciting the mirror with an voltage impulse of 25V for 2ms and fitting the decay curve towards \(\theta(t) = \theta_0 e^{-D\omega_0 t} \cos(\sqrt{1-D^2}\omega_0 t)\), we identify the linear damping coefficient \(b = 2JD\omega_0 = 5.091 \times 10^{-11} \text{Nms}\) and the small signal stiffness \(k_0 = J\omega_0^2 = J(2\pi f_0)^2\). The eigenfrequency is determined as \(f_0 = 113.3\text{Hz}\). The approximated nonlinear progressive stiffness \(k(\theta)\) taken from an Ansys finite element model with the measured linear spring stiffness \(k_0 = 1.972 \times 10^{-6} \text{Nm/rad}\) (derived from small signal behavior around zero tilt position) is sketched in Fig. 2b, and the corresponding spring torque is determined by Eq. (2):

\[
\tau_s(\theta) = \int_0^\theta k(\theta')d\theta' - k(0) + k_0.
\]

By evaluating the static voltage deflection characteristic \(\bar{\theta}(\bar{\theta}_{1,2})\), cf. Fig. 1c, we compute the nonlinear electrostatic capacitances using Eq. (3), as shown in Fig. 2a [1]. For tilt deflections where the regarding comb is not engaged, we approximate the capacitance derivative with an exponential extrapolation.

\[
C_{1,2}^{\prime}(\bar{\theta}) = 2\frac{\tau_s(\bar{\theta})}{\tau_{1,2}^{\prime}(\bar{\theta})}
\]

![Graph](image)

(a) Capacitance derivatives calculated with Eq. (3); the dashed curves are exponential extrapolations beginning from \(\theta = 0\) [1]

(b) Progressive spring stiffness from Ansys finite element model, corrected with measured small signal spring stiffness \(k_0\) around zero tilt position

Figure 2: Model parameters for capacitance and spring stiffness


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State space model

From the torque equilibrium Eq. (1) we derive the state space model Eq. (4) [3] in the form
\[ \dot{x} = f(x, u) \]
and defining a generalized input \( u \) as follows:
\[ u := \frac{J C_1'(\theta)}{2} v_1^2 + \frac{1}{2} J C_2'(\theta) v_2^2. \] (5)

Jerk-limited trajectories

Most raster scan applications require triangle trajectories or sawtooth trajectories. We apply jerk limited trajectories [1] as shown in Fig. 10, 11 for the nominal applications, because non-smooth trajectories lead to high oscillation of the very low damped spring mass system eigenmode [8]. We generate these trajectories by assembling segments of the polynomials Eq. (6), with the appropriate constants for each segment: maximum jerk \( j_{\text{max}} = \frac{\dot{\theta}_{\text{max}}}{\theta_{\text{max}}} \), start deflection \( \theta_0 \), start velocity \( \dot{\theta}_0 \) and start acceleration \( \ddot{\theta}_0 \).

\[ \theta_d(t) = \theta_0 + \dot{\theta}_0 t + \frac{\dot{\theta}_0}{2} t^2 + \frac{j_{\text{max}}}{6} t^3, \]
\[ \dot{\theta}_d(t) = \dot{\theta}_0 + \ddot{\theta}_0 t + \frac{j_{\text{max}}}{2} t^2, \]
\[ \ddot{\theta}_d(t) = \dddot{\theta}_0 + j_{\text{max}} t. \] (6)

Open-loop control

The open-loop control, cf. Fig. 3, is realized by inverting the parameterized system Eq. (4) and (5), as shown in [3]. First, we calculate the desired feedforward acceleration \( u_d \) of the generalized input by evaluating the second part of Eq. (4) with the desired trajectory \( (\theta_d, \dot{\theta}_d, \ddot{\theta}_d) \):
\[ u_d = \frac{\tau_s(\theta_d)}{J} + b \dot{\theta}_d + \ddot{\theta}_d \] (7)

Second, a switch toggles between the comb electrodes according the sign of the desired generalized input \( u_d \) as follows:
\[ v_1 = \begin{cases} \sqrt{2J u_d C_1'(\theta_d)} & \text{for } u_d > 0 \\ 0 & \text{for } u_d \leq 0 \end{cases} \]
\[ v_2 = \begin{cases} 0 & \text{for } u_d > 0 \\ \frac{2J u_d C_2'(\theta_d)}{C_2''(\theta_d)} & \text{for } u_d \leq 0 \end{cases}. \] (8)

All open-loop calculations can be processed offline.

3. CLOSED-LOOP CONTROL

The open-loop control presented in section 2 cannot compensate model inaccuracies or external disturbances, like shock or long-term parameter drifts caused by temperature and pressure changes. For this reason we develop and compare three different closed-loop control strategies feeding back an online measurement of the actual mirror tilt deflection. We realize these feedback controller complemented by our open loop control explained above.
3.1 Linear control

As linear control law, we implement the robust PID controller \[9\] given with the Eq. (9) as a Laplace transform. This method influences directly the drive voltage for controlling the position error \(e(s) = \theta_d - \tilde{\theta}\), defined as the difference between desired trajectory \(\theta_d\) and measured tilt angle \(\tilde{\theta}\), cf. Fig. 4.

\[
\Lambda(e) = \left( k_p + k_i \frac{1}{s} + k_d s \right) \frac{s}{1 + T_n s} e(s) \tag{9}
\]

Adding the controller Eq. (9) to the model-based feedforward control voltage \(v_d\) with Eq. (10), we achieve tracking the operating point with the controller correcting the open-loop model errors and external disturbances.

\[v^* = \Lambda(e) + v_d \tag{10}\]

The feedforward Eq. (11) for linear control is calculated similar to the open-loop control Eq. (8) by using Eq. (7).

\[v_d = \begin{cases} \sqrt{2J u_d} & \text{for } u_d > 0 \\ \sqrt{2J u_d} & \text{for } u_d \leq 0 \end{cases} \tag{11}\]

Finally, the electrode comb selection is realized according to the sign of the control voltage \(v^*\), Eq. (12).

\[v_1 = \begin{cases} v^* & \text{for } v^* > 0 \\ 0 & \text{for } v^* \leq 0 \end{cases} \quad v_2 = \begin{cases} 0 & \text{for } v^* \geq 0 \\ v^* & \text{for } v^* < 0 \end{cases} \tag{12}\]

![Figure 4: Linear control structure](image)

3.2 Flatness-based control

The given nonlinear system, Eq. (4), satisfies the property of differential flatness \[17, 18\], that was proved in \[3\], using the flat output \(y := x_1 = \theta\). Flat systems are a subclass of globally invertible systems \[19\]. The flatness-based nonlinear feedback control stabilizes the system state \(x\) with the position error \(e = \theta_d - \tilde{\theta}\) and the velocity error \(\dot{e} = \dot{\theta}_d - \dot{\tilde{\theta}}\), by adding the stabilizing feedback \(\Lambda(e, \dot{e})\) to the feedforward control \(u_d\). Consequently, we define the new input \(u^*\) Eq. (13):

\[u^* = \Lambda(e, \dot{e}) + u_d \tag{13}\]
The system model Eq. (4) has two independent states, that can be stabilized using the following error definition:
\[ \Lambda := k_P e + k_D \dot{e} + k_1 \int_0^t e(t')dt' \] (14)

Compared to the linear control, the nonlinear (flatness-based and sliding-mode, see next paragraph) control structures consider the electrostatic nonlinearities in the nonlinear switch using the observed mirror position \( \hat{\theta} \) with Eq. (15). The nonlinear control structure is shown in Fig. 5.

\[
v_1 = \begin{cases} \sqrt{2Ju^* / C_1(\hat{\theta})} & \text{for } u^* > 0 \\ 0 & \text{for } u^* \leq 0 \end{cases}
\]

\[
v_2 = \begin{cases} 0 & \text{for } u^* > 0 \\ \sqrt{2Ju^* / C_2(\hat{\theta})} & \text{for } u^* \leq 0 \end{cases}
\] (15)

Figure 5: Nonlinear control structure (flatness-based control and sliding-mode control)

3.3 Sliding-mode control

We implement the sliding-mode control [10, 11] Eq. (16) by adding the gain \( q \) with the sign of the hypersurface to the flatness-based controller Eq. (14). The control structure is equivalent to flatness-based control, cf. Fig. 5.

\[
\Lambda (e, \dot{e}) = k_P e + k_D \dot{e} + k_1 \int_0^t e(t')dt' + q \cdot \text{sign} \left( r_1 \int_0^t e(t')dt' + r_2 e + \dot{e} \right)
\] (16)

The hypersurface \( s(\int e, \dot{e}) \) describes a sliding surface defined with the constants \( r_1, r_2 \) on which the control errors pass along to decrease the tracking error. The sign function leads to chattering of the control output \( \Lambda \). Chattering usually results in increased actuator wear due to high frequency actuator motion. As the electrostatic comb drive operates without frictional connections, chattering does not limit the actuator functionality.

3.4 Observer design

As the angular velocity \( \dot{\theta} \) cannot be measured directly, we design an observer to estimate the system state \( \hat{x} \). The observer Eq. (17) represents a virtual copy of the system Eq. (4) applying the same inputs \( v_1, v_2 \) and the same output \( y = \theta \). The additional correction term \( K \dot{e} \) adapts the estimated system state \( \hat{x} \) towards the real measured mirror output \( \theta \) using observer gain \( K = (\kappa_1, \kappa_2)^T \) and observing error \( \hat{e} = \dot{\theta} - \dot{\theta} \).

\[
\dot{\hat{x}} = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{A} \end{pmatrix} \hat{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w + K \dot{e}
\]

\[
y = \begin{pmatrix} 1 & 0 \end{pmatrix} \dot{\hat{x}}
\] (17)
The observer dynamics is parameterized by placing the two observer poles with Eq. (18) at identical real frequencies applying the ACKERMANN formula [21].

\[
\det ([sI - (A - KC^T)]) = s^2 + \kappa_1 s + \kappa_2
\]  

(18)

We test two variants for the new input \( w \), Eq. (19), resulting in two observers: the normal form observer, Fig. 6a, using the measured deflection \( \tilde{\theta} \) and the High-Gain observer [22], Fig. 6b, using the estimated deflection \( \hat{\theta} \), for evaluating the nonlinear spring torque \( \tau_s \).

**Normal form observer:**
\[
w = \alpha(\tilde{\theta}, v_1, v_2) = u - \frac{1}{J} \tau_s(\tilde{\theta})
\]

(19a)

**High-Gain observer:**
\[
w = \alpha(\hat{\theta}, v_1, v_2) = u - \frac{1}{J} \tau_s(\hat{\theta})
\]

(19b)

Figure 6: Observer structures for normal form and High-Gain observer

The experimental and simulative comparison of both observers demonstrates a similar behavior, see Fig. 7. We find the lowest observer errors at an absolute pole frequency between 1kHz and 3kHz. However the High-Gain observer shows a smaller error for observer pole frequencies below 1kHz. Therefore, we subsequent apply the favorite High-Gain observer for the experimental validation in section 4.

Figure 7: Observer error \( \hat{e} = \hat{\theta} - \tilde{\theta} \) (absolute peak-to-peak error of 200 periods), comparing normal form and High-Gain observer with open-loop control for a 50Hz triangle trajectory as shown in Fig. 10b: measured data are illustrated with straight line and Matlab/Simulink® simulation (including noise of \( \sigma_{PSD} = 6.36 \) m°) data with dashed line.
4. EXPERIMENTAL VALIDATION

4.1 Real-time setup

The experimental validation is performed with the setup shown in Fig. 8. All online calculations are processed with 40kHz on the real-time controller DS1007 from dSPACE®, that is piloted by a host computer. The controller output signals are delivered as low voltage signal $v_1^*, v_2^*$ with the DA-converter DS2102 and then amplified with a 50x high-voltage amplifier from TEGAM® to the desired drive voltages $v_1, v_2$. For measuring the mirror tilt angle we apply the laser beam deflected by the micro scanning mirror on a position sensitive device (PSD). The PSD displacement currents are converted to the sum $v_{PSD}^+$ and difference $v_{PSD}^-$ voltages and digitalized by the AD-converter DS2004. The actual deflection angle is finally calculated using Eq. (20), where $l_{PSD}$ specifies the PSD size and $k_{PSD}$ denotes a calibration factor.

$$\tilde{\theta} = \frac{1}{2} \arctan \left( \frac{l_{PSD} v_{PSD}^-}{k_{PSD} v_{PSD}^+} \right)$$

4.2 Experimental results

First, we validate the presented control methods with a step response to $5°$, i.e. half of the maximum deflection, as shown in Fig. 9. The closed-loop setup receives a true step as an input signal. For all three closed-loop methods, we determine a valuable settling time within 0.1% of steady state deflection in about 3 ms, cf. Tab. 1 with the fastest result using sliding-mode control. The control parameter are tuned manually. For open-loop control the identical control reference will lead to excessive oscillation, as we have previously shown [8]. To enable a clear comparison we have designed a smooth step with 5 ms rise time as input signal for open loop control, using the polynomials Eq. (6). We see in Fig. 9 that the settling time is not only limited by this rise time but also by remaining oscillatory motion. The result is a settling time in the order of 500 ms.

<table>
<thead>
<tr>
<th>Control Method</th>
<th>Settling Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open-loop</td>
<td>484.9</td>
</tr>
<tr>
<td>Linear closed-loop</td>
<td>3.65</td>
</tr>
<tr>
<td>Flatness-based closed-loop</td>
<td>3.28</td>
</tr>
<tr>
<td>Sliding-mode closed-loop</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 1: Settling time for step response to $5°$ within 0.1% from Fig. 2: smooth step designed with 5 ms for open-loop control and discontinuous step for closed-loop control

For raster scanning the quasistatic axis requires performing a triangle or sawtooth shaped trajectory. While the triangle trajectory features an overall shorter reversal time, the sawtooth trajectory has a beneficial advantage in data processing concerning blurring effects, because the scan pattern keeps up the same direction [23, 24]. To better interpret the tracking performance, let us define the following errors, Eq. (21).
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Table 2: Summarized measurement results of Fig. 10,11 with peak-to-peak mean error $\Delta e$, Eq. (21b) in the linear area, for triangle and sawtooth trajectory with 10 Hz and 50 Hz and $t_{\text{max}} = 10^{-5}$.

<table>
<thead>
<tr>
<th>trajectory type</th>
<th>triangle</th>
<th>sawtooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>trajectory frequency</td>
<td>10 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>$\Delta e$ for linear control</td>
<td>0.112 %</td>
<td>0.295 %</td>
</tr>
<tr>
<td>$\Delta e$ for flatness-based control</td>
<td>0.078 %</td>
<td>0.271 %</td>
</tr>
<tr>
<td>$\Delta e$ for sliding-mode control</td>
<td>0.130 %</td>
<td>0.433 %</td>
</tr>
</tbody>
</table>

Figure 10: Measurement results for real-time control on triangle 10 Hz and 50 Hz trajectory using linear, flatness-based and sliding-mode control.
Figure 11: Measurement results for real-time control on sawtooth 10 Hz and 50 Hz trajectory using linear, flatness-based and sliding-mode control.

(a) 10 Hz sawtooth command trajectory with linear area (gray shaded) from $-9.5^\circ$ to $9.5^\circ$

(b) 50 Hz sawtooth command trajectory with linear area (gray shaded) from $-9^\circ$ to $9^\circ$

(c) Mean control error $\epsilon(t)$ of 200 periods in percent of max. deflection $\theta_{\text{max}} = 10^\circ$ for 10 Hz sawtooth trajectory (Fig. 11a) using the following parameters for linear control: $k_P = 5000, k_I = 0, k_D = 2, T_N = 1/3000$ s; for flatness-based control: $k_P = 2 \times 10^7, k_I = 0, k_D = 5000, f_{\text{obs}} = 6f_0$; for sliding-mode control: $k_P = 2 \times 10^7, k_I = 0, k_D = 5000, f_{\text{obs}} = 4f_0, q = 2000$

(d) Mean control error $\epsilon(t)$ of 200 periods in percent of max. deflection $\theta_{\text{max}} = 10^\circ$ for 50 Hz sawtooth trajectory (Fig. 11b) using the following parameters for linear control: $k_P = 2000, k_I = 0, k_D = 8, T_N = 1/4000$ s; for flatness-based control: $k_P = 4 \times 10^7, k_I = 0, k_D = 3000, f_{\text{obs}} = 6f_0$; for sliding-mode control: $k_P = 4 \times 10^7, k_I = 0, k_D = 2000, f_{\text{obs}} = 4f_0, q = 2000$


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5. SUMMARY

In this contribution we have validated the real-time closed-loop control for the quasistatic axis of a 2D micro scanner with a novel electrostatic SVC drive intending raster scan applications. In the experimental validation we achieve about 3 ms settling time for a step response to 5° deflection. Furthermore we tested triangle and sawtooth trajectories with a max. deflection of 10° featuring very low tracking errors of about 0.1 % for 10 Hz and about 0.15 % to 0.3 % for 50 Hz. Regarding the closed-loop control with an error of 0.078 % for a 10 Hz triangle trajectory, cf. Tab. 2, and the former open-loop errors 1.4079 % [1] and 1.456 % [3], we have improved the control performance by factor 18. The comparison of the linear and two nonlinear control strategies yield the best performance with the flatness-based control structure for most test cases. Nevertheless the linear control stays a relevant candidate for an analog realization, that is prospectively required for mirrors with high eigenfrequencies above 10kHz. Moreover, in the nonlinear control methods the High-Gain observer shows an advantageous behavior for small observer gains, compared to the normal form observer.

6. OUTLOOK

Integrated piezo-resistive sensors enable system operation without need for an external sensor, resulting in a system design with smaller package density and lower cost. The on-chip piezo-resistive sensor [15, 16] was depicted with high noise and unserviceable as a feedback sensor for the demonstrated device, cf. Fig. 1b. Recent developments [13] have optimized the signal quality and now will allow a feasible position feedback for the closed-loop control methods as presented in this paper.

REFERENCES


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