

Optimal Route Planning for Electric Vehicles with Special Consideration of the Topography and Battery Lifetime

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Abstract

In contrast to conventional routing systems, which determine the shortest distance or the fastest path to a destination, this work designs a route planning specifically for electric vehicles by finding an energy-optimal solution while simultaneously considering stress on the battery. After finding a physical model of the energy consumption of the electric vehicle including heating, air conditioning, and other additional loads, the street network is modeled as a network with nodes and weighted edges in order to apply a shortest path algorithm that finds the route with the smallest edge costs. A variation of the Bellman-Ford algorithm, the Yen algorithm, is modified such that battery constraints can be included. Thus, the modified Yen algorithm helps solving a multi-objective optimization problem with three optimization variables representing the energy consumption with (vehicle reaching the destination with the highest state of charge possible), the journey time, and the cyclic lifetime of the battery (minimizing the number of charging/discharging cycles by minimizing the amount of energy consumed or regenerated). For the optimization problem, weights are assigned to each variable in order to put emphasis on one or the other. The route planning system is tested for a sub-urban area in Austria and for the city of San Francisco, California. Topography has a strong influence on energy consumption and battery operation. Depending on the starting and destination point, the results are different depending on the weight of the optimization variable. Also, the tests are conducted for different outside temperatures and weather conditions, as well as for different vehicle types.

Keywords: Electric vehicles, Route planning, Yen algorithm, Multi-objective optimization, Battery lifetime

Nomenclature

The modeling and optimization variables used for the route planning system are listed below:

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Vehicle parameters	$E_{\rm drive}$	energy for driving
mass	$E_{\rm regen}$	regenerative energy from driving
drag coefficient	$E_{\rm acc}$	energy for accessories
cross sectional area	$E_{ m hc}$	energy for heating and cooling
state of charge of the battery	E_{out}	energy from the battery
width/height		Efficiencies
velocity	$\eta_{ m d}$	final drive
Road parameters	$\eta_{ m m}/\eta_{ m g}$	motor/generator
length of a road section	$\eta_{ m inv}$	DC-AC inverter
slope of a road section in $\%$	$\eta_{ m acc}$	accessories
slope of a road section in rad	$\eta_{ m hc}$	heating and cooling
External parameters	$\eta_{ m cha}/\eta_{ m dis}$	battery charging/discharging
outside temperature		Topography data
gravitational constant	R_{earth}	earth radius
air density	ϕ/ψ	coordinates in longitude/latitude
rolling resistance coefficient	$\Delta x / \Delta y$	distances
Energy consumption	Δz	height difference
rolling resistance		Multi-objective Optimization
air resistance	a	minimum battery capacity factor
gradient resistance	γ/δ	weights for optimization
driving force	$f_i^{(k)}/g_i^{(k)}/h_i^{(k)}$	costs to node i (k^{th} iteration)
power for driving	$d_{i,j}/t_{i,j}/a_{i,j}$	edge costs from i to j
power for heating	$E_{\min}/T_{\min}/A_{\min}$	optimization results
	Vehicle parameters mass drag coefficient cross sectional area state of charge of the battery width/height velocity Road parameters length of a road section slope of a road section in % slope of a road section in rad External parameters outside temperature gravitational constant air density rolling resistance coefficient Energy consumption rolling resistance air resistance gradient resistance driving force power for driving power for heating	Vehicle parameters E_{drive} mass E_{regen} drag coefficient E_{acc} cross sectional area E_{hc} state of charge of the battery E_{out} width/height η_d velocity η_d Road parameters η_m/η_g length of a road section η_{inv} slope of a road section in $\%$ η_{acc} slope of a road section in πd η_{hc} External parameters η_{cha}/η_{dis} outside temperature y_{cha}/η_{dis} gravitational constant R_{earth} air density ϕ/ψ rolling resistance coefficient $\Delta x/\Delta y$ Energy consumption Δz rolling resistance a gradient resistance a gradient resistance a driving force $f_i^{(k)}/g_i^{(k)}/h_i^{(k)}$ power for driving $d_{i,j}/t_{i,j}/a_{i,j}$ power for heating $E_{min}/T_{min}/A_{min}$

1. Introduction

Combustion engine driven cars have been dominating our world for more than a century. With global warming concerns and increasingly stringent environmental policies ahead, vehicles with electric motors could be part of the solution, a precondition being that electrical power comes from renewable sources. In addition, further challenges come up with this new technology option in transportation. Despite higher efficiency than combustion engines, the task to store energy in the vehicle is still challenging. The battery offers a much more limited range compared to conventional cars. Moreover, re-charging an electric vehicle is more time-consuming than refilling a tank. This leads to a more complex trip planning with an electric vehicle. A good advice is to identify the location of charging stations beforehand and to plan the route accordingly. Energy consumption can vary significantly depending on the path chosen. This work elaborates on optimal route planning to a desired destination while considering the special characteristics of electric vehicles. The main focus lies on improving the battery lifetime, as well as minimizing energy consumption and journey time while taking into account impacts of topography.

The first objective of this work is to find a route from a starting to a destination point with the least amount of energy used. This task is expanded such that it is possible to calculate the shortest journey time as well as the best route to maximize battery lifetime. Those route planning options are combined into a weighted multi-objective optimization problem. Shortest path algorithms are used in networks consisting of nodes, edges, and edge costs to solve the optimization problem. With the help of these algorithms, it is possible to find the path from one node to another with the smallest edge costs. Based on a model of the electric vehicle, which describes the physics of different driving modes, the energy consumption and travel time are approximated. It includes the energy required for driving as well as additional loads for air conditioning and other accessories. The model also considers engine operation as either a motor or as a generator and thus respects regenerative energy.

The following Section 2 gives an overview of relevant work covering this topic. Section 3 explains the methodology. This includes the model, which is used to calculate the vehicle's energy consumption, and the impact additional loads and efficiencies have on the vehicle. Furthermore, the street network model, the shortest path algorithm, and the multi-objective optimization are explained. Section 4 presents the results of the optimization algorithm tested for an urban as well as a sub-urban area. In addition, a sensitivity analysis investigates the effects of temperature and different vehicle models from different manufacturers. Section 5 presents conclusions and elaborates on still open questions in this field of research.

2. State of the Art

Creating an optimal route planning system for electric vehicles is multi-disciplinary and requires profound knowledge of electric vehicles, batteries, route planning algorithms, and dynamic optimization.

Modeling Electric Vehicles. Energy efficiency and battery conservation are the main goals of the proposed optimal route planning system. Therefore, the electric vehicle is described based on a physical model in order to enable calculation of the energy consumption. In this context, one special characteristic of electric vehicles is the ability to regenerate energy from braking or driving downhill. In [1] the energy flow from regenerative braking is modeled in detail. Another detailed model of the vehicle's energy consumption is presented in [2]. The goal is to find a high-resolution powertrain efficiency estimation. The energy consumption is described by the physical forces acting on the moving vehicle. A similar concept can be found in [3], where an energy consumption simulator is presented. An alternative approach to estimate the energy consumption of an electric vehicle is found in [4], where a range estimator is created by using different drive cycles, which are originally based on combustion engines and then adapted for electric vehicles. However, this approach avoided modeling the physics of an electric vehicle.

Battery Lifetime in Electric Vehicles. Having batteries as an energy storage in vehicles comes along with many new challenges compared to cars with conventional combustion engines. Apart from their costly production, batteries have a limited lifetime. According to [5], by 2020 nearly one million electric vehicle battery packs will be 'retired' in the United States alone, meaning that they cannot be used in electric vehicles anymore. This happens when (i) the battery only provides less than 80% of the original capacity and maximum power, or (ii) there are functional failures occurring. Several models to describe the battery degradation can be found in [6], where cycle aging as well as calendar aging mechanisms are considered. [7] proposes an optimal charging energy management that minimizes the degradation of the battery as a function of temperature and depth of discharge. In [8] a battery degradation model of the cycle losses is used, which is linear dependent in cumulative current throughput, but quadratic in temperature. One interesting result is found in [9], where capacity fade of commercial Li-ion batteries caused by driving and vehicle-to-grid usage is studied. Battery degradation was found to be related to the amount of energy processed in the battery, while the depth of discharge has less effects on the battery lifetime. The importance of methods to increase the battery lifetime corresponds with the work in [10], where it is shown that replacing a battery frequently may harm the environment and increases the risk of toxicity (e.g. for humans and freshwater) and metal depletion impacts.

Route Planning Algorithms. Route planning problems are typically solved by applying shortest path algorithms. The Dijkstra algorithm [11] is a very efficient method to work with weighted graphs and is widely used in network theory. A Dijkstra-based speed-up technique for constrained shortest path problems is applied in [12], where route planning is conducted for bicycles. The Dijkstra algorithm is also applied in [13]. In that work, route planning in urban areas with influences of traffic is considered. The Bellman-Ford [14] algorithm has a higher complexity than Dijkstra, but can be used in networks with negative edge costs, which is useful for route planning for electric vehicles. Despite its high complexity, the Bellman-Ford algorithm works well with street networks. The Yen algorithm [15] is based on Bellman-Ford, but it is generally able to solve the shortest path problem faster than [14].

Route Planning for Electric Vehicles. It is mentioned in [16] that many people, who tested electric vehicles, experience so-called range anxiety. It turns out that many drivers would change their driving behavior and especially their route-choices to the destination. In order to find an optimal route that can extend the range of the vehicle. Dijkstra's shortest path algorithm is applied. The goal is not to simply find the energy-minimal route, but also to help with range anxiety, making drivers feel more comfortable with e-mobility. In [17] a range estimation for on-line use is created, which is based on calculating the energy optimal route. The vehicle's energy consumption is modeled including influences of traffic and with a shortest path algorithm (Bellman-Ford) the range can be estimated. In [18] battery switch station are included in the network and a modification of Dijkstra's algorithm is used. This modification is done by using Johnson's shifting technique (see [19]) in order to include regenerated energy of the electric vehicle. In practice, battery switch stations are not established, but charging stations become increasingly visible. In [20], the approach is similar to [18], but including charging stations instead of battery switch stations. It is also proposed to work on a multi-criteria optimization that includes the journey time and a maximum number of recharging events.

The present work implements a multi-objective optimization approach determining a path that seeks to meet best the drivers' requirements. In detail, the optimization algorithm uses weighting factors on these three optimization variables:

- (i) Energy consumption: The electric vehicle should reach the destination with the highest possible state of charge of the battery.
- (ii) Time: The journey time should be as short as possible.
- (iii) Battery lifetime: In order to increase the battery lifetime of the electric vehicle, the number of charging and discharging cycles should be as small as possible, therefore the cumulated energy flow should be minimized.

The goal of this paper is to include the drivers' preferences in a very flexible way. For example, the drivers can choose a single-objective optimization of either of the above-mentioned variables, or give equal weights to all three. Charging stations for electric vehicles are not included in this work.

3. Methodology

3.1. Optimization Problem

The objective of this work is to design a route planning system for electric vehicles in order to optimize the battery lifetime, energy consumption, and journey time. A weighted multi-objective optimization approach is used to prioritize one aspect or the other.

3.2. Flowchart

To achieve the objective of this work - finding the weighted optimal path to a desired destination with an electric vehicle - it is important to have a model describing the energy consumption of the electric vehicle and the street network. Then, shortest path algorithms can be used for optimization. A flow chart (see Figure 1) presents an overview of this optimization problem. At first, a starting and a destination point have to be defined. Then, it is necessary to have a road network that includes topography information. Each road section has a defined length s, a velocity v, and a slope q in %, calculated from the topography information. The next step is to define the vehicle parameters. The initial state of charge of the battery SoC_{init} and its maximum capacity SoC_{max} are necessary, as well as the mass m, the drag coefficient c_w , and the cross sectional area A of the vehicle. When having information on the outside temperature T, all parameters are available for the calculation of the energy consumption model of the electric vehicle and the journey time on all the possible roads of the network.

As this work formulates an optimization problem with constraints, the boundary conditions have to be defined. Naturally, the state of charge of the battery SoC cannot be negative or above the maximum capacity. In the interest of the vehicle owner, the SoC should not fall below a certain minimum capacity, because deep discharge can decrease battery lifetime. A

factor $a, 0 \le a < 1$, is added to the optimization problem, such that the constraint equation becomes

$$a SoC_{\max} \le SoC \le SoC_{\max}.$$
 (1)

The task will be extended to a multi-objective optimization with the variables energy, time, and battery lifetime. The factors γ and δ , with $\gamma + \delta \leq 1$, are used to give weights to the optimization variables. The last step is to apply a shortest path algorithm.



Figure 1: Flowchart of the optimization problem

3.3. Modeling the Energy Consumption

3.3.1. Energy Consumption for Driving

We start with the calculation of the energy consumption of an electric vehicle for driving. The basic principles of the calculations are similar to conventional vehicles. The driving force F_{drive} is calculated as the sum of the rolling resistance F_{roll} , the air resistance F_{air} , and the

gradient resistance F_{grad} , while other factors are neglected for simplicity:

$$F_{\rm drive} = F_{\rm roll} + F_{\rm air} + F_{\rm grad} \tag{2}$$

$$= mgf_R\cos(\alpha) + \frac{1}{2}\rho c_w Av^2 + mg\sin(\alpha).$$
(3)

With the simplification of velocity and slope staying constant over the time $t \in [t_1, t_2]$, we have

$$P_{\rm drive} = F_{\rm drive} \cdot v, \tag{4}$$

and then finally

$$E_{\rm drive} = P_{\rm drive} \,\Delta t,\tag{5}$$

with $\Delta t = t_2 - t_1$.

3.3.2. Total Energy Consumption of the Electric Vehicle

Apart from regenerative energy from braking or driving downhill, the battery is the only source of energy in electric vehicles. Other loads than driving must be covered by the battery as well. The major consumers are the heating and cooling, which are powered directly by the high voltage battery (around 400 V), without any DC-DC converter (see [21]). Other so-called accessory loads are headlights, fan, windshield wipers, rear window heating, and radio. These need low voltage (12 V) and therefore a DC-DC converter.

Figure 2 and Figure 3 show the energy flow from the battery to the components of the vehicle with the engine operating as a motor or as a generator, respectively. E_{out} is the energy the battery provides. E_{acc} and E_{hc} represent the low voltage accessories, and the heating and cooling, respectively. E_{drive} is used for driving, as in (5). The amount of energy E that decreases the state of charge of the battery considering efficiency factors is

$$E = \frac{1}{\eta_{\rm dis}} \left(\frac{1}{\eta_{\rm d} \eta_{\rm m} \eta_{\rm inv}} E_{\rm drive} + \frac{1}{\eta_{\rm acc}} E_{\rm acc} + \frac{1}{\eta_{\rm hc}} E_{\rm hc} \right),\tag{6}$$

if the engine operates as a motor (Figure 2). With electric vehicles, recharging the battery with regenerated energy from braking or driving downhill is possible. In this case, the driving power P_{drive} in (4) is negative and the engine operates as a generator, see Figure 3. There are two cases to distinguish: (i) the battery charges, only if the regenerated energy is sufficient to cover the accessory loads; (ii) the battery discharges, if the regenerated energy cannot cover all the accessory loads. The energy $E_{\text{out}} = -E_{\text{in}}$ can be calculated as

$$E_{\rm out} = -\eta_{\rm d}\eta_{\rm g}\eta_{\rm inv}E_{\rm regen} + \frac{1}{\eta_{\rm acc}}E_{\rm acc} + \frac{1}{\eta_{\rm hc}}E_{\rm hc},\tag{7}$$

and the total energy including charging and discharging efficiencies is

$$E = \begin{cases} \frac{1}{\eta_{\text{dis}}} E_{\text{out}}, & \text{if } E_{\text{out}} > 0\\ \eta_{\text{charge}} E_{\text{out}}, & \text{if } E_{\text{out}} < 0\\ 0, & \text{if } E_{\text{out}} = 0. \end{cases}$$

$$\tag{8}$$



Figure 2: Energy flow - engine operates as a motor

If the efficiency of the engine operating as a generator is the same as for a motor $(\eta_m = \eta_g)$, Equation (6) and Equation (7) become

$$E_{\rm out} = \left(\frac{1}{\eta_{\rm d}\eta_{\rm m}\eta_{\rm inv}}\right)^{\rm sign(E_{\rm drive})} E_{\rm drive} + \frac{1}{\eta_{\rm acc}} E_{\rm acc} + \frac{1}{\eta_{\rm hc}} E_{\rm hc}.$$
 (9)

If the charging and discharging efficiencies of the battery are equal $(\eta_{cha} = \eta_{dis})$, Equation (6) and Equation (8) merge to one new equation

$$E = \left(\frac{1}{\eta_{\rm dis}}\right)^{\rm sign(E_{\rm out})} E_{\rm out},\tag{10}$$

with E_{out} as in (9).



Figure 3: Energy flow - engine operates as a generator

3.4. Modeling the Street Network

The next task of the route planning is to model the real-life street network with its properties such as slope, distances, speed limits, and road type. In order to solve a shortest path problem, a specific structure of the street network is needed.

3.4.1. Networks

Networks consist of so-called nodes and edges. The nodes are connected with each other by edges, which can have assigned values called 'edge weights' or 'edge costs'.

Nodes. Nodes are the decision points of the network. Each node is connected to different nodes via edges. Some nodes are directly connected, others are only reachable by passing other nodes on the way. In the real-world street network, nodes are intersections of roads.

Edges. Edges are the roads connecting the nodes. In this network model, the edges are split into straight sections with constant slope and constant speed limit.

Edge costs. Networks have the purpose to express relations between the nodes. There are different ways to characterize such relations. The most elementary ones are a binary relations, describing whether there is a direct connection (an edge) or not. Edge costs can have physical meanings, such as the distance between nodes. A shortest path algorithm finds the path that cumulates the least amount of edge costs on its way. When the physical meaning of the edge costs is the energy consumption of an electric vehicle, the network becomes bidirectional. The main reason is the topography, as two nodes can be on different altitudes and, therefore, the required energy is different for both directions.

3.4.2. Topography Data

Topography data is obtained from the 'USGS Earth Explorer' website [22], where geographical data is available for download. The file obtained from USGS is a TIFF-file with 3601 × 3601 pixel, containing a digital elevation map from the NASA Shuttle Radar Topography Mission (SRTM). It has a high spatial resolution of one arc-second for longitude (East-West) as well as for latitude (North-South). The height resolution is one meter. Since two adjacent pixels are one arc-second apart, the TIFF-file covers an area of one degree in longitude and one degree in latitude. For the calculations of the energy consumption of the electric vehicle, it is necessary to convert the distances given in degrees into distances in meters. The conversion is achieved by an approximation of the earth as a perfect sphere with radius of $R_{\text{earth}} = 6371 \,\text{km} = 6371000 \,\text{m}$. ϕ_1 and ψ_1 are the coordinates of a point in degree longitude and degree latitude, respectively. ϕ_2 and ψ_2 are the coordinates of another point, and $\Delta \phi = \phi_1 - \phi_2$ and $\Delta \psi = \psi_1 - \psi_2$ are the distances between point 1 and point 2 (in degrees). The distances Δx for longitude and Δy for latitude in meters are converted using

$$\Delta y = \frac{2\pi R_{\text{earth}}}{360} \,\Delta\psi,\tag{11}$$

and

$$\Delta x = \frac{2\pi R_{\text{earth}}}{360} \cos(\psi_1 \pi / 180) \,\Delta\phi. \tag{12}$$

It can be noticed that (12) depends on the coordinates of the latitude. At the equator, we have $\cos(\psi_1 \pi/180) = 1$. The coordinate lines of the longitude converge when moving further away from the equator. Two places with a $\Delta \phi$ of 1° at the equatorial line have a higher Δx than two places with the same $\Delta \phi$ at another degree of latitude. The resolution of one arc-second of the map obtained from NASA means having a resolution of about 31 m North-South. It has to be mentioned that (11) and (12) are only approximations of the real distances.

The next task is to calculate the slope of a road section that is straight and has a constant gradient. The difference in altitude Δz between the start and the end point of the section, which is derived from the TIFF-file mentioned in the beginning of this section, is divided by the euclidean distance between those points and multiplied by 100 in order to get the slope q in %:

$$q = 100 \frac{\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}.$$
(13)

3.4.3. Street Network

After obtaining topographical data, the next step is to get an adequate network of roads containing the region of interest. One option is using 'OpenStreetMaps', a big database for streets, roads, and paths. The use of 'OpenStreetMaps' is problematic because many roads are missing labels with no possibility of distinguishing if they are footpaths, waterways, or streets. The same goes for data obtained from official government websites. Thus, a more simple approach is chosen. First, it is decided which roads should be part of the network. Only the main roads in the area of interest are included by taking coordinates of road sections. Some of the roads (edges) are a few kilometers long and split up into straight consecutive sections with a length varying from 50 m to a few hundred meters. For the purpose of testing the algorithm, this resolution is satisfying. For an advanced route planning system, a higher resolution would be needed. The coordinates of the start and end points of the road segments and the matching topography information obtained, the distances and height differences are calculated as explained in Section 3.4.2. This ensures each segment is straight and has a constant slope.

3.5. Optimization with a Shortest Path Algorithm

The multi-objective optimization is solved by applying a shortest path algorithm. Shortest path algorithms use networks made of nodes and edges. The goal of shortest path algorithms is to find the path with the least amount of edge costs. In this case the edge costs represent the energy consumption, journey time, or the battery degradation. Section 3.5.1 explains the shortest path algorithm for one optimization variable. Then, the multi-objective optimization based on the shortest path algorithm is described in Section 3.5.2.

3.5.1. Yen Algorithm

A well known shortest path algorithm is the Bellman-Ford algorithm, see [14]. An improved version of Bellman-Ford is the algorithm proposed by Jin Y. Yen, see [15]. Both

algorithms find the shortest paths from all nodes of the network to one specific destination node. The iterative search goes backwards, always starting from the destination node. In contrast, the state of charge of the battery (SoC) is calculated forward, starting from the starting point. The constraints are violated if the SoC exceeds the boundaries, see (1). In addition, the goal for energy-optimal route planning is to arrive at the destination with the highest state of charge possible. The backwards iterations of the Yen algorithm are, therefore, not suitable for the optimization problem with SoC-constraints. Hence, the following solution is introduced. Instead of computing all minimum paths to the destination, all minimum paths from the start are calculated. The principles of the Yen algorithm remain the same, but there are a few modifications. The algorithm is applied to find the energyoptimal path in a network with N nodes in the following manner, starting with iteration 0,

$$f_i^{(0)} = d_{1i},\tag{14}$$

where d_{1i} are the edge costs from node 1 to node i = 2, ..., N, representing the energy consumption required to travel from node 1 to i. $f_i^{(k)}$ are the costs from node 1, the start node, to node i at iteration k. A distinction is made between odd and even numbered iterations. For odd iterations, the minimum is found using

$$f_i^{(2k-1)} = \min_{1 \le j < i} (f_j^{(2k-1)} + d_{ji}, f_i^{(2k-2)}),$$
(15)

$$f_1^{(2k-1)} = f_1^{(2k-2)}, (16)$$

with $i = 2, 3, \ldots, N$. For even iterations, the minimization is

$$f_i^{(2k)} = \min_{N \ge j > i} (f_j^{(2k)} + d_{ji}, f_i^{(2k-1)}),$$
(17)

$$f_N^{(2k)} = f_N^{(2k-1)},\tag{18}$$

with i = N - 1, N - 1, ..., 2. For all iterations, the battery constraints are checked. If $SoC > SoC_{\max}$, the path is still feasible, but the current state of charge is set to the maximum capacity of the battery ($SoC = SoC_{\max}$), because no further charging is possible. The energy consumption $f_j^{(k)} + d_{ji}$ is adapted accordingly. If $SoC < a SoC_{\max}$, that path is considered unfeasible and $f_i^{(k)}$ is set to infinity, such that this path is neglected in all further calculations. If all minimum paths f_i of iteration k remain the same compared to the previous iteration k - 1, the optimum is found:

$$\boldsymbol{f}^{(k)} = \begin{bmatrix} f_1^{(k)} \\ f_2^{(k)} \\ \vdots \\ f_N^{(k)} \end{bmatrix} \equiv \begin{bmatrix} f_1^{(k-1)} \\ f_2^{(k-1)} \\ \vdots \\ f_N^{(k-1)} \end{bmatrix} = \boldsymbol{f}^{(k-1)}.$$
(19)

For time-optimal route planning, the optimum $g_i^{(k)}$ of each iteration k is calculated having the edge costs t_{ji} , the journey time from node j to node i (see Appendix A). Similarly, the

optimum for the battery lifetime $h_i^{(k)}$ works with the edge costs a_{ji} (see Appendix B). a_{ji} represents the absolute energy flow to and from the battery.

3.5.2. Multi-objective Optimization

The multi-objective optimization has three optimization variables: energy consumption of the vehicle, journey time, and cyclic lifetime of the battery. The algorithm applied for this task is based on the modified version of the Yen algorithm from Section 3.5.1. The optimal path from the start node number 1 to all nodes i is calculated in each iteration, taking into account all three variables and their weights, as well as the battery constraints. We start with iteration 0, setting all three variables in the same manner:

$$f_i^{(0)} = d_{1i},\tag{20}$$

$$g_i^{(0)} = t_{1i},\tag{21}$$

$$h_i^{(0)} = a_{1i}.$$
 (22)

The next step is to go into the details of the odd iterations. At first,

$$f_1^{(2k-1)} = f_1^{(2k-2)}, (23)$$

$$g_1^{(2k-1)} = g_1^{(2k-2)}, (24)$$

$$h_1^{(2k-1)} = h_1^{(2k-2)} \tag{25}$$

are set. Then, for the other nodes *i*, the energy optimal path is calculated following (15). If at least one path is feasible, $E_{min} = |f_i^{(2k-1)}|$ is set and T_{min} , the minimum in time, and A_{min} , the smallest absolute energy consumption, are calculated according to (A.4) and (B.2), respectively. All three results are combined to multi-objective optimization with the weights γ for energy, δ for time, and $1 - (\gamma + \delta)$ for battery lifetime. The optimization problem solved for node $i = 2, 3, \ldots, N$ in iteration 2k - 1 is:

$$x_{i}^{(2k-1)} = \min_{1 \le j < i} \left(\gamma \, \frac{f_{j}^{(2k-1)} + d_{ji}}{E_{min}} + \delta \frac{g_{j}^{(2k-1)} + t_{ji}}{T_{min}} + (1 - (\gamma + \delta)) \frac{h_{j}^{(2k-1)} + a_{ji}}{A_{min}}, \right.$$

$$\gamma \, \frac{f_{j}^{(2k-2)}}{E_{min}} + \delta \frac{g_{j}^{(2k-2)}}{T_{min}} + (1 - (\gamma + \delta)) \frac{h_{j}^{(2k-2)}}{A_{min}} \right).$$

$$(26)$$

All energy costs are normalized by the factor E_{min} , the journey time by T_{min} , and the absolute energy by A_{min} . The normalized values equal to one at the optimum and greater than one for all the other paths. The deviation from the optimum is weighed with γ , δ and $1 - (\gamma + \delta)$. After the minimum $x_i^{(2k-1)}$ is found, the costs $f_i^{(2k-1)}$, $g_i^{(2k-1)}$, and $h_i^{(2k-1)}$ are set according to the resulting multi-objective optimal path. For even iterations the situation is very similar. We start with

$$f_N^{(2k)} = f_N^{(2k-1)},\tag{27}$$

$$g_N^{(2k)} = g_N^{(2k-1)}, (28)$$

$$h_N^{(2k)} = h_N^{(2k-1)}. (29)$$

Then, for node *i* the optimal values E_{min} , T_{min} , and A_{min} are computed and multi-objective optimization is performed, iterating from i = N - 1 to i = 2:

$$x_{i}^{(2k)} = \min_{N \ge j > i} \left(\gamma \, \frac{f_{j}^{(2k)} + d_{ji}}{E_{min}} + \delta \frac{g_{j}^{(2k)} + t_{ji}}{T_{min}} + (1 - (\gamma + \delta)) \frac{h_{j}^{(2k)} + a_{ji}}{A_{min}}, \right.$$

$$\gamma \, \frac{f_{j}^{(2k-1)}}{E_{min}} + \delta \frac{g_{j}^{(2k-1)}}{T_{min}} + (1 - (\gamma + \delta)) \frac{h_{j}^{(2k-1)}}{A_{min}} \right).$$
(30)

3.6. Reference Values and Assumptions

The two vehicles selected for testing are the Nissan Leaf, see data sheet [23], and the Mitsubishi i-MiEV, see data sheet [24]. All parameters necessary for the calculations are summarized in Table 2. For the mass m, the curb weight of the vehicle, is combined with the weight of one person, the driver. The driver's mass is assumed to be 75 kg. To calculate the air resistance F_{air} according to (3), the cross sectional area A and the drag coefficient c_w are necessary. If A is not specified in the official data sheet, it may be estimated according to [25], knowing the width w and height h of the vehicle:

$$A = 0.81 \cdot w \cdot h. \tag{31}$$

Accessory loads and efficiencies are obtained from the measurements in [26]. Some efficiency factors cannot be found in [26], therefore they are assumed to be 100%, except for the engine efficiency, which is assumed to be 90%.

		Nissan Leaf	Mitsubishi i-MiEV	
Dimensions				
Curb weight		1516	1090	kg
Mass with driver	m	1591	1165	kg
Drag coefficient	c_w	0.28	0.33	-
Width with mirrors		1967	1792	$\mathbf{m}\mathbf{m}$
Width w/o mirrors	w	1770	1475	$\mathbf{m}\mathbf{m}$
Height	h	1550	1610	mm
Cross sectional area	A	2.22	2.14	m^2
Accessory loads				
Heating	P_{heat}	90	95	W/°C
Cooling	$P_{\rm cool}$	40	30	Ŵ∕°C
Lights	P_{light}	48	38/127	W
Fan	P_{air}	62	48	W
Battery				
Capacity	SoC_{\max}	24	16	kWh
Efficiencies				
Drive	$\eta_{ m d}$	1	1	
Motor	$\eta_{ m m}$	0.90	0.90	
Inverter DC-AC	$\eta_{ m inv}$	0.96	0.91	
Battery	$\eta_{ m dis}$	0.90 - 0.96	0.88 - 0.95	
Accessories DC-DC	$\eta_{ m acc}$	1	0.83	
Heating/Cooling	$\eta_{ m hc}$	1	1	

Table 2: Parameters of the two different electric vehicles

4. Results

4.1. Suburban Area: Near Vienna, Austria

The street network covers the main roads of the Wienerwald area West of Vienna. The city of Vienna is excluded. The network has a total of 31 nodes and 41 edges. Different routes were tested with the multi-objective optimization algorithm. Planning a trip from Passauerhof to Maria Gugging provides interesting results (see Figure 4). There are two different solutions of the multi-objective optimization. The routes are:

- (a) The first path, where the electric vehicle needs the least amount of energy, starts at Passauerhof, and passes through Katzlsdorf, Königstetten, and St. Andrä before it reaches the destination Maria Gugging (see Figure 4).
- (b) The other route (b) is both the fastest and the shortest route, but presents more variation in topography than (a), passing small villages such as Unterkirchen and Hintersdorf before reaching Maria Gugging (see Figure 4).



Figure 4: Results from Passauerhof to Maria Gugging (edited screen-shot from Google Maps; Google and the Google logo are registered trademarks of Google LLC, used with permission.)

Table 3 presents the solutions of the multi-objective optimization for various combinations of the weights. It can be noticed that combinations with small values of γ and δ result in option (a). This route has less variation in topography than (b), as it can be seen in Figure 5 and 6. If γ and δ are small, the focus is on increasing the battery lifetime. In contrast to the optimization variable for energy consumption, where the regenerated energy has a negative sign and, therefore, helps minimizing the costs, the optimization variable representing the battery lifetime takes the absolute value of the energy. In this case, the regenerated energy counts the same way as the consumed energy. Therefore, driving downhill or braking increases the costs and is avoided by the optimization algorithm. A route with little elevation up and down is preferred. It can be seen in Figure 6 that the absolute energy flow to and from the battery for route (a) is smaller than for route (b). Combinations

γ/δ	0	0.2	0.4	0.6	0.8	1
0	(a)	(a)	(a)	(b)	(b)	(b)
0.2	(a)	(a)	(a)	(b)	(b)	-
0.4	(a)	(a)	(b)	(b)	-	-
0.6	(a)	(a)	(b)	-	-	-
0.8	(a)	(b)	-	-	-	-
1	(a)	-	-	-	-	-

Table 3: Results from Passauerhof to Maria Gugging with Nissan Leaf at 20 $^{\circ}\mathrm{C}$

with a high value of δ lead to the fastest route (b). The difference in time is almost four minutes, which is a quarter of the total journey time of route (b). The difference in energy consumption is less significant (see Figure 5). Choosing route (a), only a few Watt-hours are saved. If the battery lifetime is neglected ($\gamma + \delta = 1$), a high value of γ and a small δ lead to route (b), while only with $\gamma = 1$, route (a) is chosen, because the savings in energy consumption are insignificant compared to the savings in journey time.



Figure 5: Topography and energy consumption on route (a) and route (b) from Passauerhof to Maria Gugging

4.1.1. Comparing Different Weather Conditions

The route planning system is also tested for different outside temperatures. During winter, the temperatures frequently fall below zero in Austria. Therefore, heating is necessary for the passengers in the vehicle. With the help of the measurements in [26], it is possible to find a linear function of the Nissan Leaf's power demand for heating, which is $P_{\text{heat}} = 90 \text{ W/}^{\circ}\Delta \text{C}$. The energy consumption is expected to rise with lower temperatures,



Figure 6: Topography and absolute energy consumption on route (a) and route (b) from Passauerhof to Maria Gugging

which may influence the results of the multi-objective optimization problem. Assuming an outside temperature of $T = -10^{\circ}$ C, we have

$$P_{\rm hc} = P_{\rm heat}(T_0 - T) = 1800 \,\mathrm{W},\tag{32}$$

with $T_0 = 20^{\circ}$ C. For the relatively short trip of around 20 minutes, 0.6 kWh are used only for heating. In addition, battery (dis-)charging efficiency η_{dis} is decreasing due to low temperatures (see Table 2). Table 4 shows the results for varying combinations of the optimization weights. These results are not surprising considering that the energy consumption of all

				(a) 20	0°C				(b) -10°C						
γ	δ / δ	0	0.2	0.4	0.6	0.8	1]	γ/δ	0	0.2	0.4	0.6	0.8	1
	0	(a)	(a)	(a)	(b)	(b)	(b)		0	(a)	(a)	(b)	(b)	(b)	(b)
0).2	(a)	(a)	(a)	(b)	(b)	-		0.2	(a)	(a)	(b)	(b)	(b)	-
0).4	(a)	(a)	(b)	(b)	-	-		0.4	(a)	(b)	(b)	(b)	-	-
0).6	(a)	(a)	(b)	-	-	-		0.6	(b)	(b)	(b)	-	-	-
0).8	(a)	(b)	-	-	-	-		0.8	(b)	(b)	-	-	-	-
	1	(a)	-	-	-	-	-		1	(b)	-	-	-	-	-

Table 4: Results from Passauerhof to Maria Gugging different outside temperatures

accessories, including heating and cooling, is time-dependent. This factor makes fast routes more attractive also when looking at energy-optimal route planning. In this case, route (a), which has less energy consumption at 20 °C, now requires more energy than route (b), making route (b) the most energy and time efficient route at temperatures of -10 °C outside (see Figure 7). When focusing on the cyclic lifetime of the battery only (low γ and δ), route (a) is still chosen because of the topography, as explained previously. Figure 8 shows that the cumulated absolute energy consumption and regeneration of route (a) is still smaller than that of route (b), but with a narrower margin at -10 °C compared to 20 °C.



Figure 7: Energy consumption (cumulated) on route (a) and route (b) from Passauerhof to Maria Gugging for different outside temperatures, 20 °C and -10 °C



Figure 8: Absolute value (cumulated) of the energy consumed or regenerated on route (a) and route (b) from Passauerhof to Maria Gugging for different outside temperatures, $20 \,^{\circ}C$ and $-10 \,^{\circ}C$

4.1.2. Comparing Two Different Types of Electric Vehicles

So far, the tests are performed exclusively with the Nissan Leaf. In the following part, the Mitsubishi i-MiEV vehicle is added, assuming its characteristics as given in Table 2. The Mitsubishi vehicle is almost 30% lighter in weight than the Nissan, but its c_w -value is higher and its efficiencies are lower. In Table 5, the results for the Nissan Leaf and Mitsubishi i-MiEV are compared. Figure 9 shows that the energy consumption on both routes, (a)

(a) Nissan Leaf										(b)	Mitsubi	shi i-Mi	EV	
γ/δ	0	0.2	0.4	0.6	0.8	1		γ/δ	0	0.2	0.4	0.6	0.8	1
0	(a)	(a)	(a)	(b)	(b)	(b)]	0	(a)	(a)	(a)	(b)	(b)	(b)
0.2	(a)	(a)	(a)	(b)	(b)	-		0.2	(a)	(a)	(b)	(b)	(b)	-
0.4	(a)	(a)	(b)	(b)	-	-		0.4	(a)	(a)	(b)	(b)	-	-
0.6	(a)	(a)	(b)	-	-	-		0.6	(a)	(a)	(b)	-	-	-
0.8	(a)	(b)	-	-	-	-		0.8	(a)	(b)	-	-	-	-
1	(a)	-	-	-	-	-]	1	(a)	-	-	-	-	-

Table 5: Results from Passauerhof to Maria Gugging at 20 $^{\circ}\mathrm{C}$

and (b), is higher for the Mitsubishi despite having less weight. Another finding is that on both routes the cumulated absolute value of the consumed and regenerated energy of the Mitsubishi is smaller compared to Nissan. This result shows that the Mitsubishi regenerates less energy. Only once does the result change compared to the Nissan, which is for $\gamma = 0.2$ and $\delta = 0.4$. The absolute energy has the weight of $1 - (\gamma + \delta) = 0.4$. In this case, the objective to be minimized in (26) is almost equal for both routes, but with route (b) being slightly more efficient.



Figure 9: Energy consumption (cumulated) on route (a) and route (b) from Passauerhof to Maria Gugging comparing Nissan Leaf and Mitsubishi i-MiEV at 20 $^{\circ}\mathrm{C}$



Figure 10: Absolute value (cumulated) of the energy consumed or regenerated on route (a) and route (b) from Passauerhof to Maria Gugging comparing Nissan Leaf and Mitsubishi i-MiEV at $20 \,^{\circ}\text{C}$

4.2. Urban Area: San Francisco, California

Further tests are conducted for the city of San Francisco, California, because of its interesting topography. The street network of San Francisco is fundamentally different from the street network of Wienerwald, Austria. It is an urban area with a high road density. The network is a grid of perpendicular streets. Each intersection represents a node of the network. Because of the high road density, there is also a high number of nodes. Therefore, only a small part of downtown San Francisco is selected in order to keep the network more compact.

Modeling an urban street network may be challenging because there are many factors that are hard to predict. Traffic flow is on top of the agenda in this context. When searching for the fastest route, the current traffic situation and the timing of traffic lights are main concerns, other factors among many others may be being zebra crossings, stop signs, and bus stops. They not only influence the journey time, but also the energy consumption. Nevertheless, the impacts of traffic and road signs are neglected for simplification in the present work. The focus is on the energy consumption and battery lifetime, which are expected to be considerably impacted by the topography of the city. Multi-objective optimization is performed with $\delta = 0$.

4.2.1. Route: From Pier 39 to Russian Hill

Different starting and destination points are tested. One journey starts close to Pier 39, at the corner Beach Street/Grant Avenue. The destination point is Union Street/Hyde Street, close to Russian Hill. All combinations of the multi-objective optimization weights result in the same route (see Figure 11). Going in the opposite direction, from Russian Hill to Pier 39, gives the result seen in Figure 11, for all values of γ . Figure 13 (dashed lines) shows the topography of the two scenarios: Going from Pier 39 to Russian Hill (Union/Hyde Street) and the return. The energy consumption on both trips (see Figure 13) strongly de-



Figure 11: Results from Pier 39 to Russian Hill (screen-shot from Google Maps - Google and the Google logo are registered trademarks of Google LLC, used with permission.)



Figure 12: Results from Russian Hill to Pier 39 (screen-shot from Google Maps - Google and the Google logo are registered trademarks of Google LLC, used with permission.)

pends on the topography. Since Pier 39 is almost at sea level, going to Russian Hill takes a lot of energy, while the electric vehicle regenerates energy by going in the opposite direction down to Pier 39. In Figure 14, it can be seen that the absolute energy flow of the return trip is a lot smaller.

Further results of San Francisco show that focusing on the battery lifetime often results in



Figure 13: Topography and energy consumption from Pier 39 to Russian Hill and back



Figure 14: Topography and absolute energy consumption from Pier 39 to Russian Hill and back

routes differing from those focusing on energy efficiency. Switching the destination point to another one close to the original results in the three routes represented in Figure 15. Figure

16 and Figure 17 show the topography as well as the energy consumption and absolute energy consumption of the three results. Route (a) is the result for $\gamma = 0$ and $\gamma = 0.2$. It is longer and not as energy efficient as route (b) and (c), but shows better performance in terms of battery-lifetime due to a different topography. Route (c) is the most energy efficient route ($\gamma = 0.8$ and $\gamma = 1$).



Figure 15: Route options (a), (b), and (c) in San Francisco (edited screen-shot from Google Maps; Google and the Google logo are registered trademarks of Google LLC, used with permission.)



Figure 16: Topography and energy consumption for route options (a), (b), and (c) in San Francisco



Figure 17: Topography and absolute energy consumption for route options (a), (b), and (c) in San Francisco

5. Conclusion and further Developments

This work shows that route planning specifically designed for electric vehicles has more to offer than conventional routing systems, which only consider time *or* distance to the destination. The multi-objective optimization uses three optimization variables: energy consumption, journey time, and battery lifetime. With different weights on the variables, different solutions are obtained. Tests are performed for existing street networks. In some cases, there is only one optimal route for all variables, while other cases show very distinct results for one of the optimization variables.

The results demonstrate the influence of the topography on the routes. The energy consumption depends significantly on the topography. Especially with electric vehicles, which may regenerate energy by driving downhill, there is a strong correlation between energy consumption and topography. Another aspect of this work is optimization in order to increase the battery lifetime. Minimizing the total energy flow of the battery is the goal and, therefore, routes with little topographical variation are preferred. The battery is an expensive part of the vehicle and, as such, should be protected from degradation as much as possible. The influence of additional loads such as heating, air condition, lights, and fan is significant. Those loads are powered by the battery and therefore increase the total energy consumption, with an important factor being the duration of the journey, as more energy is used the longer these additional loads operate. Accordingly, the results of the multi-objective optimization may change, due to fast routes becoming more energy efficient.

Using a weighted multi-objective optimization may be more reasonable than using a singleobjective optimization. If the second fastest route were much more energy efficient, but only slightly slower than the fastest route, a time-only optimization leads to an unreasonable result, but when different aspects are considered at the same time, the most practical solution is achieved. In order to use this type of route planning for real-world scenarios, some additional information may be necessary. Traffic lights and stop signs should be included, as well as traffic flow, as a theoretically optimal route may in practice prove to be infeasible due to traffic holdups. Future work on this topic should also include charging stations such that route planning is possible for a journey farther than the battery range of the vehicle.

Lastly, another important aspect to be included in future work is driving behavior. Speed and acceleration influence the energy consumption of the vehicle. In urban areas, the journey time is mainly dependent on the traffic situation rather than the speed, while in non-urban areas the speed may have a significant impact. Assuming a certain speed profile could help achieving realistic results. Either a statistical approach to obtain a speed profile, or learning from previously obtained data of the driver's preferences could be utilized.

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Appendix A. Time-optimal Route Planning

Compared to the search for an energy-optimal path, finding a time-optimal path is a lot less complicated. On one hand, there are no constraints assumed concerning the journey time. It is just about finding the fastest itinerary. On the other hand, there are no negative edge costs. In fact, the graph is undirected if it is assumed that the same road with the same speed limit is available for the way back. This optimization problem could be solved with a less complex and faster algorithm than Bellman-Ford or Yen, because there are no negative edge costs. Although, when energy and time are combined to multi-objective optimization, it is easier to just use the same algorithm, because the computation time is not crucial in this work. The procedure is the same as for finding an energy-optimal path. In this case the edge costs represent the journey time. For any road segment, they are calculated as

$$t = v/s, \tag{A.1}$$

with the assigned speed level v and the length s of the section. The journey time of a whole edge connecting the nodes i and j is the sum of all M road segments:

$$t_{ij} = \sum_{l=1}^{M} t_l. \tag{A.2}$$

Iteration 0 is given as

$$g_i^{(0)} = t_{1i},$$
 (A.3)

where t_{1i} are the edges costs from node 1 to node i = 2, ..., N. For odd iterations, the minimum is found using

$$g_i^{(2k-1)} = \min_{1 \le j < i} (g_j^{(2k-1)} + t_{ji}, g_i^{(2k-2)}),$$
(A.4)

$$g_1^{(2k-1)} = g_1^{(2k-2)}, \tag{A.5}$$

with i = 2, 3, ..., N. For even iterations, the minimization is

$$g_i^{(2k)} = \min_{N \ge j > i} (g_j^{(2k)} + t_{ji}, g_i^{(2k-1)}),$$
(A.6)

$$g_N^{(2k)} = g_N^{(2k-1)},$$
 (A.7)

with $i = N - 1, N - 1, \dots, 2$.

Appendix B. Energy-optimal Route Planning in Order to Increase Battery Lifetime

The third criterion, which may be used for route planning is increasing battery lifetime. Battery wear-off should be avoided for as long as possible, as it can decrease the capacity as well as limit the range of the vehicle. The battery is also a very expensive part of the electric vehicle to replace. There are some actions to be taken in order to increase the battery lifetime. First, avoiding deep discharge is helpful. This can be included in the factor a, which can be tuned to a value that is safe for the battery.

A lithium-ion battery, which is the power source for most electric vehicles, usually has a specific cyclic lifetime. Decreasing the total number of cycles of the battery helps to increase its lifetime. This is the third criterion of the multi-objective optimization. It is performed in a similar way as the energy-optimal planning, but takes the absolute value of the energy as the edge costs. Power provided by the battery for driving or for accessories is treated the same as regenerated energy from braking and driving downhill, thus increasing the costs instead of decreasing them. The edge costs from node i to j is a_{ij} , the sum of the absolute energy values of all road sections belonging to this edge. h_i are the costs from node i.

Iteration 0 is given as

$$h_i^{(0)} = a_{1i}, \tag{B.1}$$

where a_{1i} are the edges costs from node 1 to node i = 2, ..., N. For odd iterations, the minimum is found using

$$h_i^{(2k-1)} = \min_{1 \le j < i} (h_j^{(2k-1)} + a_{ji}, h_i^{(2k-2)}), \tag{B.2}$$

$$h_1^{(2k-1)} = h_1^{(2k-2)},$$
 (B.3)

with $i = 2, 3, \ldots, N$. For even iterations, the minimization is

$$h_i^{(2k)} = \min_{N \ge j > i} (h_j^{(2k)} + a_{ji}, h_i^{(2k-1)}),$$
(B.4)

$$h_N^{(2k)} = h_N^{(2k-1)},$$
 (B.5)

with $i = N - 1, N - 1, \dots, 2$.