Performance Investigation of Angle of Arrival based Localization

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***Abstract*—By increasing the deployment of beamforming capabilities in mobile communication networks the user direction becomes readily available at base stations (BSs). Utilizing the angles of arrival (AOAs) from multiple BSs enables the calculation of the user position within one additional step using triangulation techniques.**

In our work we state the mathematical problem and derive the associated Cramér-Rao lower bound (CRLB). Further on we compare a recently proposed linear LS estimator with a trivial triangulation technique and the corresponding CRLB in three simulation scenarios.

***Index Terms*—Localization, Triangulation, CRLB, Least Squares, Massive MIMO**

I. Introduction

The topic of localization in mobile networks has been discussed based on different measurement strategies such as time of arrival (TOA) [1], received signal strength (RSS) [2] and angle of arrival (AOA) [3]. Besides the application of mentioned techniques, studies on their theoretical limits have been conducted [4], [5].

Future mobile communication networks will extend their operation into the millimetre-spectrum. In order to utilize millimetre-waves, a high antenna gain into the line-of-sight (LOS) direction is required and as a consequence beamforming into the user direction becomes mandatory. This results in future mobile communication networks having to know the AOAs for signals travelling from the user to the base stations [6].

Readily available AOA data at the base stations turns the triangulation approach into a feasible alternative to previously favoured trilateration techniques [1]. In order to fully employ AOA-based methods the performance of utilized techniques has to be investigated.

After an introduction to our work in Section I, in Section II we define the statistical problem and derive the associated Cramér-Rao lower bound (CRLB) in Section III. Further on in Section IV we present a trivial estimation technique and a direct linear LS estimator [7]. In Section V we compare them to the corresponding CRLB in three simulation scenarios. We conclude our results in Section VI.

II. Model

A. Geometric Scenario

The positions of the base stations are known. Since the base stations are capable of beamforming the respective AOA

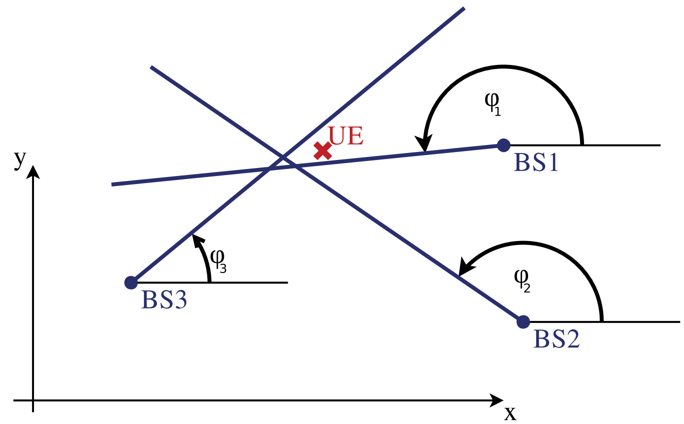

Fig. 1. Geometric setup of a generic triangulation problem.

is assumed to be available. The objective is to estimate the user location by utilizing this data. The resulting geometric setup is depicted in Figure 1. In order to simplify notation, two-dimensional problems are studied. An extension to three-dimensional structures is straight forward.

B. Statistical Model

The base station positions $(x_m, y_m), m = 1, 2, \ldots, M$ and the actual user position $\boldsymbol{\theta} = (x_u, y_u)$ are modelled as deterministic parameters hence the resulting AOAs $\varphi_m = \arctan\left(\frac{y_u - y_m}{x_u - x_m}\right)$ are also deterministic.

The estimation error of the AOA at the respective base station is modelled by adding zero-mean Gaussian noise $\mathrm{n}_m \in \mathcal{N}(0, \sigma_m^2)$ to the deterministic AOA resulting in

$$\mathrm{r}_m = \varphi_m + \mathrm{n}_m \in \mathcal{N}(\varphi_m, \sigma_m^2). \quad (1)$$

C. Measure of Performance

The performance of the estimation methods is compared in terms of the achieved root mean squared error (RMSE) performance

$$\mathrm{RMSE}_X = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (\hat{x}_n^{(X)} - x_u)^2 + (\hat{y}_n^{(X)} - y_u)^2}, \quad (2)$$

which is calculated utilizing the user position estimates $\hat{\boldsymbol{\theta}}_n^{(X)} = \left(\hat{x}_n^{(X)}, \hat{y}_n^{(X)}\right), n = 1, 2, \ldots, N$ from estimator X.

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III. Cramér-Rao lower bound

To assess the estimator's performance a theoretical bound namely the CRLB associated with the stated statistical model in (1) is derived. As the AOAs at the base stations are modelled deterministically the classical CRLB is applied [8].

The base station positions are statistically independent from each other thus the AOAs at the respective base stations are also independent, allowing the Fisher information matrix to be calculated as a sum over all base station matrices

$$\boldsymbol{J}(\boldsymbol{\theta}) = \sum_{m=1}^{M} \boldsymbol{J}_m(\boldsymbol{\theta}), \tag{3}$$

with

$$\boldsymbol{J}_m(\boldsymbol{\theta}) = \frac{1}{\sigma_m^2} \frac{1}{\left((y_u - y_m)^2 + (x_u - x_m)^2\right)^2} \begin{bmatrix} (y_u - y_m)^2 & -(y_u - y_m)(x_u - x_m) \\ -(y_u - y_m)(x_u - x_m) & (x_u - x_m)^2 \end{bmatrix}. \tag{4}$$

The desired classical CRLB is obtained as

$$\mathrm{RMSE}_X \geq \sqrt{\mathrm{tr}\left(\boldsymbol{J}^{-1}(\boldsymbol{\theta})\right)} \tag{5}$$

for any unbiased estimator X.

IV. Estimators

Two estimators are used in our work. The basis of comparison are the data sets (x_m, y_m, r_m) generated according to the aforesaid model in Section II.

A. Trivial Triangulation Estimator

This estimator is an intuitive but primitive method that is obtained by calculating beam intersection points from data sets of two base stations (x_k, y_k, r_k) and (x_l, y_l, r_l), permuting through all $\frac{(M-1)M}{2}$ combinations and averaging over them, resulting in

$$\begin{aligned} \hat{x}^{(TR)} &= \frac{2}{(M-1)M} \sum_{k=1}^{M-1} \sum_{l=k+1}^{M} \hat{x}_{k,l} \\ \hat{y}^{(TR)} &= \frac{2}{(M-1)M} \sum_{k=1}^{M-1} \sum_{l=k+1}^{M} \hat{y}_{k,l} \end{aligned} \tag{6}$$

$$\begin{aligned} \hat{x}_{k,l} &= \frac{y_l - y_k + \tan(\mathrm{r}_k)x_k - \tan(\mathrm{r}_l)x_l}{\tan(\mathrm{r}_k) - \tan(\mathrm{r}_l)} \\ \hat{y}_{k,l} &= y_k + \tan(\mathrm{r}_k)\ (\hat{x}_{k,l} - x_k). \end{aligned} \tag{7}$$

B. Linear LS Estimator

The more advanced estimator from [7] takes a linear LS approach that determines the point of minimum distance $(\hat{x}, \hat{y})$ to the beams resulting from the data sets of all base stations $(x_m, y_m, r_m), m = 1, 2, \ldots, M$ as the solution of the linear equations

$$\begin{aligned} &\begin{bmatrix} \sum_{m=1}^{M} A_{2,m} & -\sum_{m=1}^{M} A_{1,m} \\ -\sum_{m=1}^{M} A_{1,m} & \sum_{m=1}^{M} A_{0,m} \end{bmatrix} \begin{bmatrix} \hat{x}^{(LS)} \\ \hat{y}^{(LS)} \end{bmatrix}, \\ &= \begin{bmatrix} -\sum_{m=1}^{M} A_{1,m} L_m \\ \sum_{m=1}^{M} A_{0,m} L_m \end{bmatrix} \end{aligned} \tag{8}$$

$$\begin{aligned} A_{p,m} &= \frac{\tan^p(\mathrm{r}_m)}{1 + \tan^2(\mathrm{r}_m)}, \\ L_m &= y_m - \tan(\mathrm{r}_m)x_m. \end{aligned} \tag{9}$$

V. Simulations

The simulations we present focus on finding rules for well performing base station constellations and questioning the advantage of using a non-trivial estimator. All simulation scenarios are calculated with $N = 1\,000\,000$ MC runs and standard deviations $\sigma_m = 0.5^\circ : \forall m = 1, 2, ..., M$.

A. Simulation Setups

1) Initial Scenario: For the first simulation the base station and user positions are set up along two intersecting lines according to Figure 2a). This scenario serves as common ground for interpretation of the two following simulations. The movement of the user is along the red line of Figures 2a), 3a) and 4a).

2) Repositioning Base Station: The second scenario is a modification of the first one where one base station is moved away from the previously common line. As a result we expect to remove the singularity occurring at $x = -100\,\mathrm{m}$ in the first scenario visible in Figure 2b).

3) Changing Number of Base Station: Based on the first setup (Figure 2a)) in our last analysis the number of base stations is varied considering $M = \{2, 3, 4\}$ base stations. We expect to see a crucial difference in the performance of the two estimators.

B. Interpretation of Results

As expected the RMSEs at $x = -100\,\mathrm{m}$ collectively include a singularity since positions on the base station's common line can only be guessed.

Comparing Figure 2b) and Figure 3b) it can be observed that spreading the base stations gives better performance. The singularity in the RMSE of the CRLB and the linear LS method are removed; the one belonging to the trivial estimator only gets narrower.

As expected for $M = 2$ base stations the LS estimator gives no advantage over the trivial triangulation method. However at an increased number of base stations, the RMSE of the linear LS estimator improves while the performance of the trivial triangulation method gets even worse around the $x = -100\,\mathrm{m}$ position, as can be seen in Figure 4b).

Furthermore our studies show that a good assessment of the linear LS method is acquired by merely analysing the CRLB. The bad performance of the trivial triangulation method might arise from the multiple $\tan$-terms in the estimation formula in (7).

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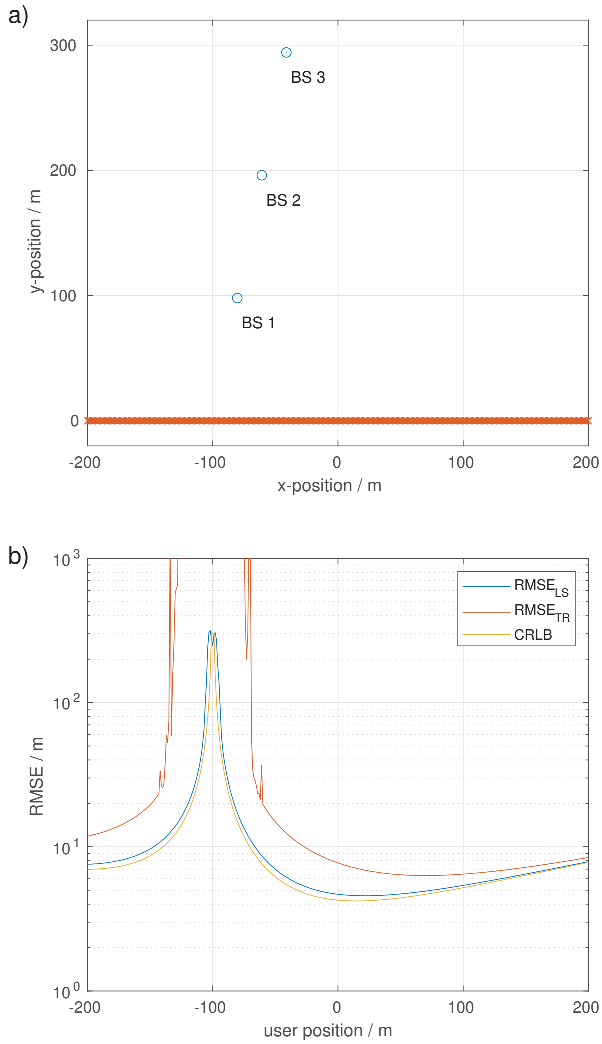

Fig. 2. a) Initial simulation scenario composed of two lines with an angle of $\arctan(5)$ between them. b) Simulation results of initial scenario, comparing LS method (LS), simple triangulation method (TR) and Cramér-Rao lower bound (CRLB).

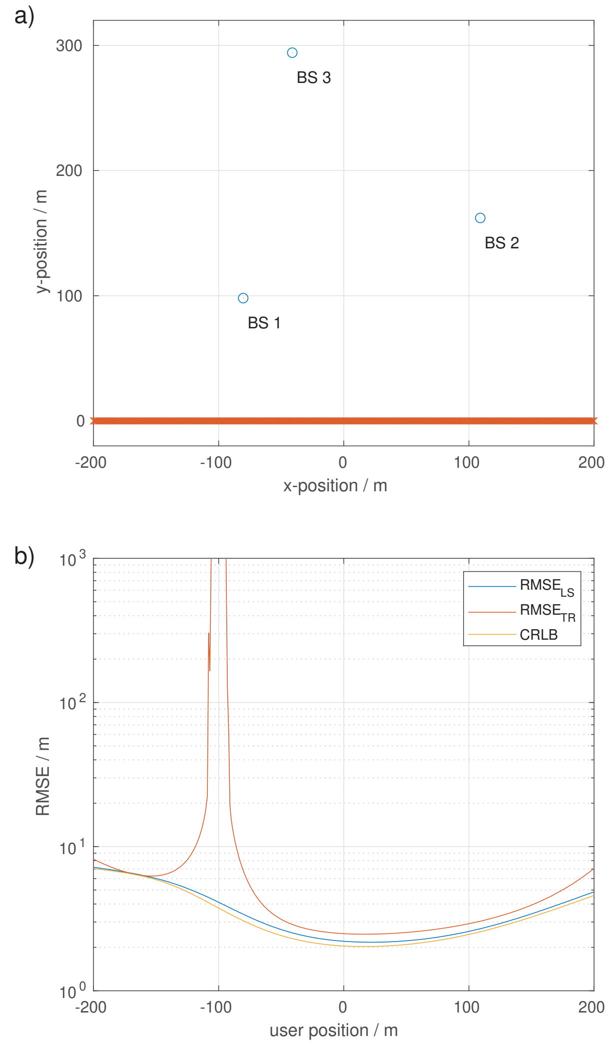

Fig. 3. a) Scenario with repositioned BS2, same distance between base stations. b) Simulation results of second scenario, comparing LS method (LS), simple triangulation method (TR) and Cramér-Rao lower bound (CRLB).

VI. Conclusion

We considered two extreme scenarios: one with base station aligned and one with a rich separation. Using the derived CRLB we showed that for a triangulation based localization setting up base stations on a straight line results in a high positioning error for users along said line. More promising results are obtained when base stations are placed further apart from each other. In both cases the CRLB is very tight to the achieved results by an LS estimator of moderate complexity, demonstrating its advantage over a primitive triangulation approach.

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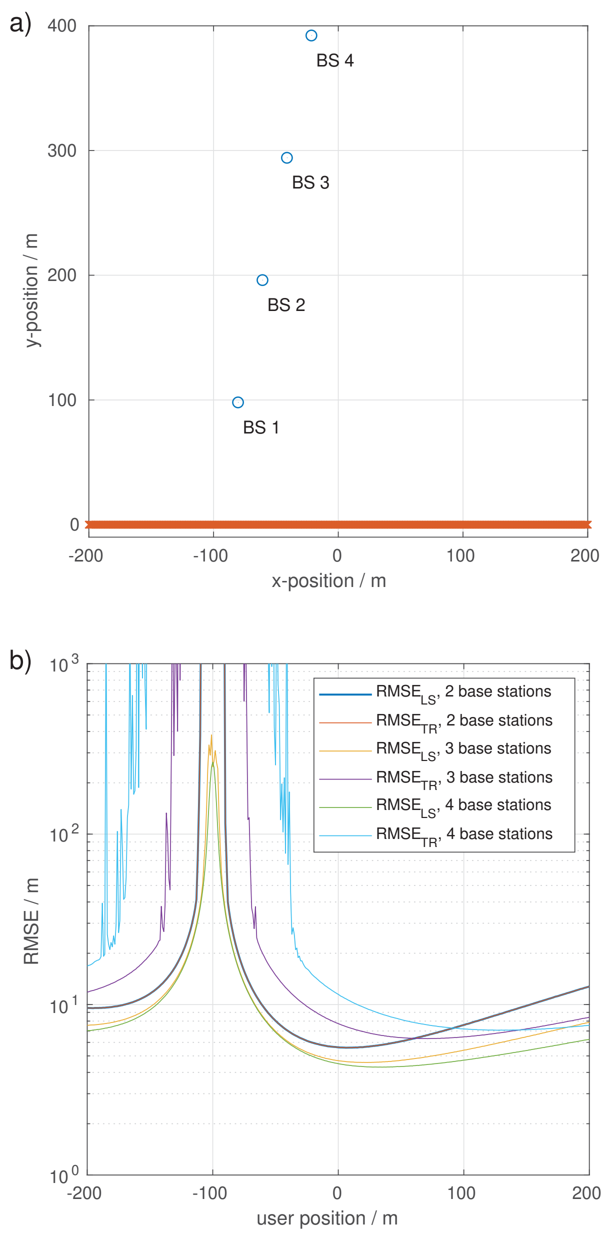

Fig. 4. a) Scenario with two, three and four base stations on a straight line. b) Simulation results of third scenario, comparing LS method (LS) with simple triangulation based estimator (TR).

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