Towards Data-Driven Multilinear Metro Maps

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Abstract. Traditionally, most schematic metro maps as well as metro map layout algorithms adhere to an *octolinear* layout style with all paths composed of horizontal, vertical, and 45°-diagonal edges. Despite growing interest in more general *multilinear* metro maps, generic algorithms to draw metro maps based on a system of $k \geq 2$ not necessarily equidistant slopes have not been investigated thoroughly. We present and implement an adaptation of the octolinear mixed-integer linear programming approach of Nöllenburg and Wolff (2011) that can draw metro maps schematized to any set $\mathcal{C}$ of arbitrary orientations. We further present a data-driven approach to determine a suitable set $\mathcal{C}$ by either detecting the best rotation of an equidistant orientation system or by clustering the input edge orientations using a k-means algorithm. We demonstrate the new possibilities of our method in a real-world case study.

1 Introduction

Metro maps are ubiquitous schematic network diagrams that aid public transit passengers in orientation and route planning in almost all types of urban public transit systems worldwide. Since Henry Beck's classic schematic London Tube Map of 1933, metro maps have developed a common visual language and adopted similar design principles. Designing professional metro maps is still mostly a manual task today, even if cartographers and graphic designers are supported by digital drawing tools. Algorithms for automated layout of metro maps have received substantial interest in the graph drawing and network visualization communities as well as in cartography and geovisualization over the last 20 years [9,14]. The vast majority of metro map layout algorithms focus on so-called *octolinear* (sometimes also called *octilinear*) metro maps, which are limited to Henry Beck's classical and since then widely adopted 45°-angular grid of line orientations [4]. However, not all metro maps found in practice are strictly octolinear. There is empirical evidence from usability studies that the best set of line orientations for drawing a metro map depends on different aspects of the respective transit network, and it may not always be an octolinear or even an equiangular one [12,13].

In this paper we present an algorithmic approach using global optimization for computing (unlabeled) metro maps in the more flexible *k-linearity* setting,

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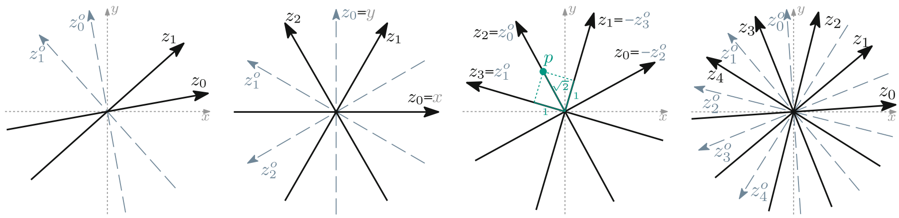
where each edge in the drawing must be parallel to one of $k \geq 2$ equidistant orientations whose pairwise angles are multiples of $360°/2k$. In this sense, a k-linear map for $k = 4$ corresponds to the traditional octolinear setting. In fact, most octolinear maps use a horizontally aligned orientation system. It is possible though, for some transit networks and city geometries, that a rotation of the orientation system by an angular offset yields a more topographically accurate metro map layout. Hence we also consider such *rotated* k-linear maps. In addition to equiangular k-linear orientation systems, we further study irregular *multilinear* (or $\mathcal{C}$*-oriented*) maps [12], in which the edges are parallel to any given, not necessarily equiangular set $\mathcal{C}$ of orientations. There exist a number of metro map layout algorithms (see [9,14,15] for comprehensive surveys) that would technically permit an adaptation to a different underlying angular grid, yet most previous papers optimize layouts in the well-known octolinear setting only and do not discuss extensions to different linearities explicitly. A few algorithms for generic multilinear or k-linear layouts exist [1,2,5,6], but they are aimed at paths or polygons rather than entire metro maps. In the field of graph drawing many algorithms for planar orthogonal network layouts with $k = 2$ as well as for polyline drawings with completely unrestricted slopes are known [3], but they do not generalize to k-linearity and multilinearity.

Contributions. We present two approaches for deriving suitable, data-dependent linearity systems (Sect. 3). Then we adapt the octolinear mixed-integer linear programming (MIP) model of Nöllenburg and Wolff [10] by generalizing their mathematical layout constraints to k-linearity and multilinearity (Sect. 4). The main benefit of this model in comparison to other approaches is that it defines sets of hard and soft constraints and guarantees that the computed layout satisfies all the hard constraints and (globally) optimizes the soft constraints. The trade-off for providing such strong quality guarantees is that computation time is typically higher compared to other methods [15]. By modeling fundamental metro map properties such as strict adherence to the given linearity system and topological correctness as hard constraints, we obtain layouts that satisfy these layout requirements strictly. The soft constraints optimize for line straightness, compactness, and topographicity [11], i.e., low topographical distortion. Our modifications yield a flexible MIP model, whose complexity measured by the number of variables and constraints grows linearly with the number of orientations k. We demonstrate the effect of horizontally aligned and rotated k-linear and multilinear orientation systems in a case study with the metro map of Vienna and evaluate the resulting number of bends and angular distortions for typical small values of $k = 3, 4, 5$ (Sect. 5).

Due to space constraints, some details are omitted; these can be found in [7].

2 Preliminaries

We reuse the notation of Nöllenburg and Wolff [10]. The input is represented as an embedded planar metro graph $G = (V, E)$ with n vertices and m edges. Each



(a) $k = 2$, irregular (b) $k = 3$, aligned (c) $k = 4$, regular (d) $k = 5$, regular

Fig. 1. Coordinate axes for different orientation systems. (c) includes a point with the redundant coordinates $p = (0, 1, \sqrt{2}, 1)$.

vertex $v \in V$ represents a metro station with x- and y-coordinates and each edge $e = (u, v) \in E$ is a segment linking vertices u and v that represents a physical rail connection between them. Finally, $k \geq 2$ is an input parameter that defines the number of available edge orientations in the orientation system $\mathcal{C}$. The set $\mathcal{C}$ and the parameter k can be part of the input or they can be derived automatically from the input geometry, see Sect. 3. Figure 1 shows three examples of orientation systems. Since every orientation can be used in two directions this yields $2k$ available drawing directions. Let $\mathcal{K}$ be this set of $2k$ directions. We note that every edge is assigned exclusively to an outgoing direction of its incident vertices, which implies that the maximum degree Δ of G can be at most $2k$. In turn, Δ gives a lower bound on the required number of orientations.

The general algorithmic metro map layout problem studied in this paper is to find a $\mathcal{C}$*-oriented schematic layout* of G, i.e., a graph layout that preserves the input topology, uses only edge directions parallel to an orientation from $\mathcal{C}$, and optimizes a weighted layout quality function (here composed of line straightness, topographicity, and compactness). If $\mathcal{C}$ corresponds to a k-linear orientation system, we also call the layout k*-linear* instead of $\mathcal{C}$-oriented; otherwise it can alternatively be called *multilinear*.

3 Orientation Systems

A set of edge orientations (or an *orientation system*) $\mathcal{C} = \{c_1, \ldots, c_k\}$ is a set of k angles (expressed in radian), where $0 \leq c_1 < \cdots < c_k < \pi$. We distinguish three different kinds of possible edge orientation sets. An edge orientation set $\mathcal{C}$ is called *regular* (or *equiangular*) if the angles $\{c_1, \ldots c_k\}$ divide the range $[c_1, c_1 + \pi)$ into k parts of equal size π/k, i.e., $c_i - c_{i-1} = \pi/k$ for all $i \in \{2, \ldots, k\}$. Otherwise we call $\mathcal{C}$ *irregular*. A regular orientation system $\mathcal{C}$, in which $c_1 = 0$ is called *aligned*. A classical octolinear layout has the orientation system $\mathcal{C}_o = \{0, \pi/4, \pi/2, 3\pi/4\}$.

Regular (non-aligned) and irregular systems allow us to derive a suitable system $\mathcal{C}$ from the geometric properties of the input data, with the goal to minimize the topographic distortion of the layout compared to the input.

We measure the distortion of $\mathcal{C}$ with respect to a metro graph G by summing up the difference in slope between each edge $e \in E$ (with slope $\gamma_e \pmod{\pi}$) and the angle $c \in \mathcal{C}$ which is closest to γ_e as $\operatorname{dist}_G(\mathcal{C}) = \sum_{e \in E} \left(\min_{c \in \mathcal{C}} |c - \gamma_e|\right)$.

3.1 Regular Orientation Systems

Fixing a single angle in a regular orientation system $\mathcal{C}$ fixes all other orientations. It is therefore sufficient to specify the first orientation $c_1 \in \mathcal{C}$. We denote by $\mathcal{C}_{\text{opt}}$ a regular orientation system with minimal distortion, i.e., $\operatorname{dist}_G(\mathcal{C}_{\text{opt}}) \leq \operatorname{dist}_G(\mathcal{C})$ for any k-regular orientation system $\mathcal{C}$. We can show [7] that one can find such an optimal system $\mathcal{C}_{\text{opt}}$, in which at least one $c \in \mathcal{C}_{\text{opt}}$ is parallel to an input edge. Thus we can restrict our search to orientation systems in $\mathfrak{C}(E) = \{\mathcal{C} \mid \exists e \in E : \gamma_e \in \mathcal{C}\}$, i.e., to orientation systems, where at least one orientation coincides with the slope of an edge in E. The set $\mathfrak{C}(E)$ contains $O(|E|)$ elements and we select $\mathcal{C}_{\text{opt}}$ as the one yielding the minimum $\operatorname{dist}_G(\mathcal{C})$ for all $\mathcal{C} \in \mathfrak{C}(E)$.

3.2 Irregular Orientation Systems

In an irregular orientation system $\mathcal{C}$ with k orientations, each orientation can be selected independently. We interpret the orientation system as a clustering of the set $\Gamma = \{\gamma_e \mid e \in E\}$ of all input edge slopes, where each cluster is formed around the closest orientation in $\mathcal{C}$. Our goal is to find a set $\mathcal{C}$ of k orientations (clusters) that minimizes $\operatorname{dist}_G(\mathcal{C})$. To this end we apply the exact 1-dimensional k-means clustering algorithm of Nielsen and Nock [8] to the set Γ. This algorithm has running time $O(n^2 k)$ using a precomputed auxiliary matrix as a look up table.

4 MIP Model

Next we sketch how the MIP model of Nöllenburg and Wolff [10] must be modified in order to compute more general $\mathcal{C}$-oriented metro maps for an arbitrary set $\mathcal{C}$ of k orientations. Hard constraints encode properties of a layout which can not be violated. Soft constraints model the aesthetic quality criteria to be optimized in the layout. The hard constraints of the MIP comprise four aspects: $\mathcal{C}$-oriented coordinate system, assignment of edge directions, combinatorial embedding, and planarity. The soft constraints comprise line straightness, topographicity, and compactness. Each requires a set of linear constraints and a corresponding linear term in the objective function. While almost all constraints require smaller modifications, we focus here only on the coordinate system as the most central change from the octolinear MIP model [10]. For our full MIP model see [7].

Coordinate System. Every vertex u of G has two Cartesian coordinates in the plane $\mathbb{R}^2$, specified as $x(u)$ and $y(u)$. In order to address vertex coordinates in an octolinear system, Nöllenburg and Wolff [10] defined a redundant system of four coordinates. To adapt this system for any number k of orientations, we define a system of k coordinates $z_0, \dots, z_{k-1}$, which are all real-valued variables in the MIP model and can all be obtained by rotating the x-axis counterclockwise by one of the angles in the orientation system $\mathcal{C} = \{\theta_0, \dots, \theta_{k-1}\} \subset [0, \pi)$. We define the coordinate $z_i(u)$ using $x(u)$ and $y(u)$ as $z_i(u) = \cos(\theta_i) \cdot x(u) + \sin(\theta_i) \cdot y(u)$.

In order to express that two vertices u, v are collinear on a line with a slope in $\mathcal{C}$, we need the orthogonal orientation z_i^o for each coordinate z_i. Note that while z_i^o *can* coincide with other coordinates, this is guaranteed only in a regular orientation system with an even number of orientations. In general, this is not the case and hence we define a second set of redundant coordinates, see Figs. 1a, 1b and 1d. Using a rotation by $\pi/2$ we obtain $z_i^o(u) = -\sin(\theta_i) \cdot x(u) + \cos(\theta_i) \cdot y(u)$.

All other constraints of [10] need to be adapted to comply with the newly created coordinate system. For a full description of the modifications see [7].

5 Experiments

We performed experiments on real-world data to compare the computational performance and visual quality of metro maps with different linearity systems. Due to space constraints, we only present the results for the metro network of Vienna. The full experimental evaluation can be found in [7].

5.1 Setup

We generated schematic layouts of the metro network of Vienna ($n = 90$, $m = 96$), using aligned, regular and irregular orientation systems with $k \in \{3, 4, 5\}$ orientations. All layouts were created with two different weight vectors $(f_1, f_2, f_3) = (3, 2, 1)$ and $(10, 5, 1)$ for the objective function[1]. For all layouts we added planarity constraints on demand and concentrated on the overall layout geometry and interchanges without showing the individual stops along the lines.

To judge the quality and performance of a layout, we use several measurements. Firstly, the total number of bends in a layout as a measure of line straightness. Secondly, the MIP allows an edge to be drawn in the direction closest to its input direction (preferred) or one direction offset to the left or right (penalized in the objective function). The sector deviation is a coarse measure of topographicity, counting how many edges are not drawn in their preferred direction. Sector deviation is measured in total and on average per edge. Another measure of topographicity is the angular distortion, i.e., the actual angular difference between input edges and schematized output edges, which is measured on average per edge. Finally, we measure the runtime in seconds.

The experiments were run as single threads on an Intel Xeon E5-2640 v4, 2.40 GHz, with 64 GB of available memory space, using IBM ILOG CPLEX 12.8.

[1] f_1 emphasizes line straightness, f_2 the topographicity and f_3 compactness of an optimal layout.

5.2 Results

The performance and quality measurements for the 18 different instances are given in Table 1. Due to space constraints, we show only one representative set of nine layouts for Vienna in Fig. 2 and omit the other nine layouts.

Table 1. Results for the Vienna network. The model parameters are the number of available directions (k) and the orientation system (ori. sys.). The measures are the number of bends, sector deviation total (sec. dev.) and per edge (p.e.), distortion per edge (dist. p.e.) and the runtime in seconds.

Weights	$k =$	3			4			5		
	Ori. sys.	Aligned	Regular	Irregul.	Aligned	Regular	Irregul.	Aligned	Regular	Irregul.
(3, 2, 1)	#bends	**16**	**16**	17	22	24	21	25	25	29
	sec. dev.	27	27	**13**	21	18	17	24	24	21
	↳ p.e.	0.28	0.28	**0.14**	0.22	0.19	0.18	0.25	0.25	0.22
	dist. p.e.	31.47	36.07	15.96	23.18	22.96	16.07	19.45	26.68	**14.46**
	time [s]	308	349	**8**	108	116	299	69	217	113
(10, 5, 1)	#bends	16	16	**15**	19	19	19	25	25	29
	sec. dev.	25	25	**19**	27	27	**19**	24	24	23
	↳ p.e.	0.26	0.26	**0.2**	0.28	0.28	**0.2**	0.25	0.25	0.24
	dist. p.e.	31.18	33.5	17.76	25.19	23.53	16.35	19.45	26.68	**15.01**
	time [s]	53	39	**8**	140	115	41	44	27	51

We refer to specific sets of instances by their number of orientations k or their weights (f_1, f_2, f_3). Our first observation from generalizing the octolinear MIP model [10] is that the model size, i.e., the numbers of constraints and variables, scales linearly with k. So as long as k is a (small) constant, the asymptotics with respect to the graph size remain the same. Yet, in practice, doubling the size of the model may yield a significant slow-down in the solution time.

Next we look at the visual effects of increasing k. The increase in bends can be explained in part by an increase in unavoidable bends. The probability that two consecutive edges in a metro line cannot be drawn in the same direction decreases with increasing k; it could be counteracted by allowing more than three sectors for each edge. Sector deviation increases (under an irregular orientation system), but for aligned and regular systems, no trend emerges. Distortion seems to decrease overall, since the maximally possible angle distortion for each edge decreases. We would expect a greater runtime for an increasing k, however we did not observe this for Vienna.

Next we compare the two different weight vectors. Unsurprisingly, we have a similar or smaller amount of bends, when emphasizing bend minimization by changing $f_1 = 3$ to $f_1 = 10$. This also (slightly) increases the angle distortion. Sector deviation is on average slightly smaller for $(3, 2, 1)$, where choosing the preferred sector is more emphasized relative to the line straightness. The more emphasized setting $(10, 5, 1)$ leads overall to lower runtimes.

Finally we compare the effect of different orientation systems. While the number of bends is comparable for aligned and regular systems, for the irregular system they increase for $k = 5$, which might be specific to Vienna [7]. Sector deviation is again comparable for aligned and regular system but improves in the irregular setting. The same is true and even more pronounced for the distortion.

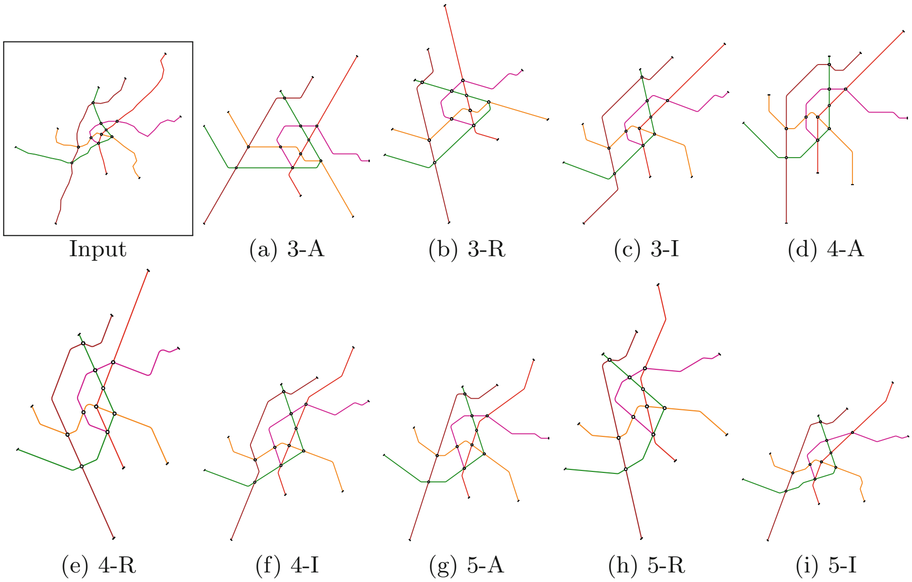


Fig. 2. Layouts of Vienna generated with objective function weights $(f_1, f_2, f_3) = (3, 2, 1)$. For each $k \in \{3, 4, 5\}$ layouts are labeled as created with aligned (k-A), regular (k-R) and irregular (k-I) orientation system.

5.3 Discussion

Our approach of increasing topographicity in metro maps through data-driven selection of orientation systems seems to be promising based on our initial experiments. Choosing an irregular orientation system is a valid option to increase topographicity, even if the irregular set of slopes is unfamiliar.

Looking at the actual metro maps produced by our system, we can see one major caveat of our approach to minimize distortion by deciding the directions based on the input. While for most edges we have a very suitable representative direction in the orientation system, the constraints of the MIP might still force an edge to be drawn in a different sector, thus working against the topographicity. On a positive note, we can see that irregular orientation systems can create metro maps that resemble the input more closely than typical aligned systems.

We can also see that most of the layouts, which are not using an aligned orientation system do not include the horizontal direction. This might be helpful in labeling these metro maps, since it is difficult to place the visually preferred horizontal labels along a horizontal line with clear association to a station.

We conclude by reinforcing that our system should not be understood as a stand-alone method to metro map generation, but rather as an automated tool to help a designer explore the layout space more thoroughly and find a suitable orientation system for a network at low time cost. One approach to choose a suitable linearity k for a given input might be to use the smallest k which generates visually appealing layouts.

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