FRACAM: A 2.5D Fractal Analysis Method for Facades

Test Environment for a Cell Phone Application to Measure Box Counting Dimension

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Fractal analysis helps explaining and understanding architectural quality, e.g., regarding visual complexity described by fractal (box counting) dimension. FRACAM, a cell phone application, uses fractal image analysis methods and takes into account the specific requirements of architectural purposes at the same time. It was developed by the authors to measure the fractal dimension of buildings; more precisely, to measure (color or grayscale) images of (street) views. This paper examines the results of various implemented algorithms for dependencies on camera settings and environmental factors. The main contribution of the authors deals with both an improved differential box counting mechanism applied to color images and a discussion about measurement results concerning influences on the algorithms presented.

Keywords: *cell phone application, box counting, fractal dimension, visual complexity, elevation analysis*

MOTIVATION

This paper is part of a broader research topic that aims to describe and understand architectural quality. According to Georg and Dorothea Franck (2008), the assessment of the quality of a building depends on several criteria, including the relationship to its surroundings, functional aspects, and sensual charisma. The main question of this research is whether measurement methods can give value to some criteria. The authors published numerous papers about particularly one aspect: the fractal characteristics concerning visual complexity, irregularity, and proportion. This contribution describes additional methods that seek to give a holistic analysis based on fractal methods.

The hypothesis is that architectural quality can be linked with fractal characteristics, including roughness, self-similarity, and scale-invariance. Under this aspect, the authors examine especially (street) views concerning visual complexity, irregularity, and proportion (Wurzer et al. 2017, Lorenz et al. 2017, Kulcke et al. 2016, Kulcke et al. 2015, Lorenz 2012, Lorenz 2013, Lorenz 2004). The main argument of these researches is the importance of architectural continuity across multiple scales. In more detail, the authors argue that at any distance from the building, the presence of design components corresponding to the human scale of the observer is essential (see also Salingaros 2006). This means that, while a person approaches a building, the building should always offer some details to look at, which could not be recognized in the previous more distant position (Lorenz 2003). From every position, elements should be visible that cover a range of different scales corresponding to man - from the entire body to the finger's width (Salingaros 2006). Regarding the concept of (statistically) self-similarity, this allows a building to remain interesting at all levels, from far to near. The linkage between all elements might be a basic shape, such as the variation of an angle in Bruce Goff's Joe Price Studio in Bartlesville; a basic idea, such as the "striving for heaven" of a gothic cathedral, reflected in arches and the entire building; or a basic proportion system, commonly used by architects throughout history. While self-similarity, the linking part, is difficult to measure, there are efforts to deal with

- proportions and redundancy, which reduces the complexity of an objects' appearance (Kulcke et al. 2016, Kulcke et al. 2015)
- proportions and grouping of architectural elements in the sense of construct/constituent pairing similar to quantitative linguistics (Lorenz et al.)
- visual complexity and the use of box counting applied to facades, represented as black and white images (Ostwald et al. 2016, Lorenz 2012)

Each of these cases addresses a different approach and uses different methods. *FRACAM*, the cell phone application described in this paper addresses particularly fractal analysis methods to measure binary black and white, grayscale, and color images.

So far, the fractal analysis in architecture has mainly focused on the box counting of two-dimensional representations of facades. Box counting applied to facades estimates the fractal dimension to give a quantitative value of visual complexity (Bovill 1996). The proposed method by the authors goes one step further and calculates the fractal dimension directly from grayscale and, for the first time, color photographs of buildings. As regards the implementation, particular consideration was given to evaluation in real-time and correctness of the results (see section 'Analyses of Case Studies'). The presented application combines various fractal analysis methods, which are necessary for a holistic consideration, and provides, at the same time, some color analysis methods, including average color, prominent colors, and color counts. Color analysis methods are particularly important for evaluating the relationships between fractal dimension and image characteristics.

This paper describes, on the one hand, the implementation of the algorithms, especially the first architectural application of the improved box counting method (Liu et al. 2014). On the other hand, it discusses case studies under the aspect of camera-specific settings and possible environmental influences. Case studies include specially prepared color images and photographs of iconic architectural examples.

METHOD

Box counting is a fractal analysis method that turned out to be a proper way for measuring the visual complexity of buildings, especially of facades (Bovill 1996, Ostwald et al. 2016). Concerning architecture, however, only the algorithm that works on two-dimensional line graphics has been used so far. To consider only the two-dimensional representation, in terms of an elevation view of the building, it also means either to discuss the design intention of the architect (if the original plan by the architect is analyzed) or the simplified representation of the reality. While the former may require previous preparation - to transform the origin into a black and white line-graphics (or vector graphics) -, the latter runs the risk of interpretation of what might be of importance for visual perception or for the design concept itself.

Apart from difficulties caused by the preparation of a measurable representation, the computer programs used, so far, are scientific local desktop implementations that may not be freely available to everyone and sometimes slow, or applications that are limited to binary and grayscale digital images (e.g. [1,2,3]). Therefore, the authors have devised a cell

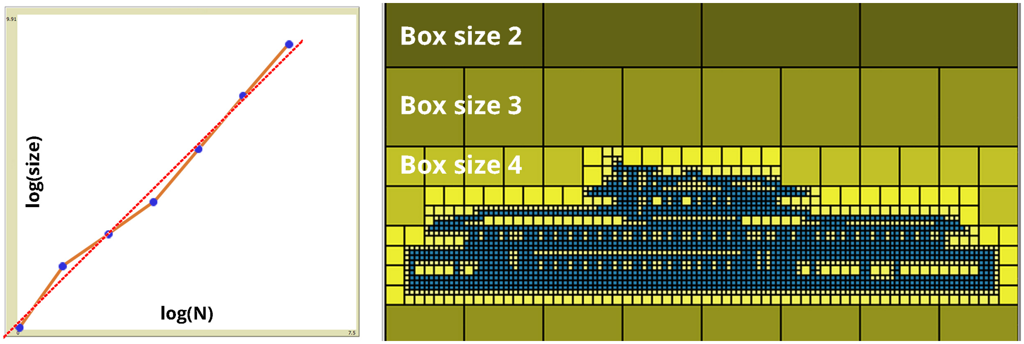

Figure 1
Left: The slope of the regression line (in a double logarithmic graph with the number of covering boxes versus the scale of the grid) gives the box counting dimension, which equals the fractal dimension. Right: The number of covering boxes in relation to the grid size (shown in different colors). This example was programmed with NetLogo (Wilensky, U. 1999).

phone application that lets users quickly grasp and analyze color photos taken with the cell phone camera (Wurzer and Lorenz 2017). In this paper, the authors describe a further development of this particular application, including a modified method for grayscale images and a method for color images. Moreover, case studies are discussed in terms of their camera settings and color characteristics.

Box counting

With the "standard" box counting, at first, a grid of an initial scale is placed over the considered two-dimensional black and white image. Thereby, the side length of a single box (cell) defines the scale of the grid. Subsequently, all boxes are counted that cover the image; in other words, those boxes are counted that contains a black part (pixel, line, or area) of the black and white image (see Figure 1 right). In the next stage, the scale is reduced, and covering boxes are counted again. This is repeated several times until the size of the boxes in the grid is too small to identify new details; that means, the procedure stops before the size becomes as small as the "empty" space between black lines. Finally, the result is visualized in a double-logarithmic graph with log(scale of the grid) versus log(number of covering boxes). The shape of the curve provides information about the dependence between the scale of the grid and the number of covering boxes. More precisely, the range of an almost straight data curve signifies the area of a strong positive correlation between both values (see Figure 1 left). Such a relationship suggests that, for this particular range of scales, a similar irregularity exists. The characteristic value, called the box counting dimension (see formula 1) is, finally, determined by the slope of the regression line.

$$D_B = \lim_{s \to 0} \frac{\log(N_s)}{\log\left(\frac{1}{s}\right)} \tag{1}$$

with N_s being the number of covering boxes, and s being the side length of a single box that defines the scale of the grid. Put simply, box counting takes into account the changing number of covering boxes of a grid depending on the changing scale of the grid, or more precisely, on the side length of a single box in the grid.

The resulting box counting dimension is adequate to the fractal dimension, which in turn is an indicator of the roughness or irregularity of natural and artificial objects. Since its first application in the field of architecture (Bovill 1996) box counting (modified or as is) was used to analyze elevation views of a building (Zarnowiecka 2002, Capo 2004, Md Rian 2007, Ostwald et al. 2016). However, in most of the cases, the utilization of two-dimensional monochrome images makes a previous process necessary. This includes the vectorized redrawing of plans, the rework of existing CAAD drawings, or edge detection for scanned published plans. Moreover, the base method does not analyze the real built ob-

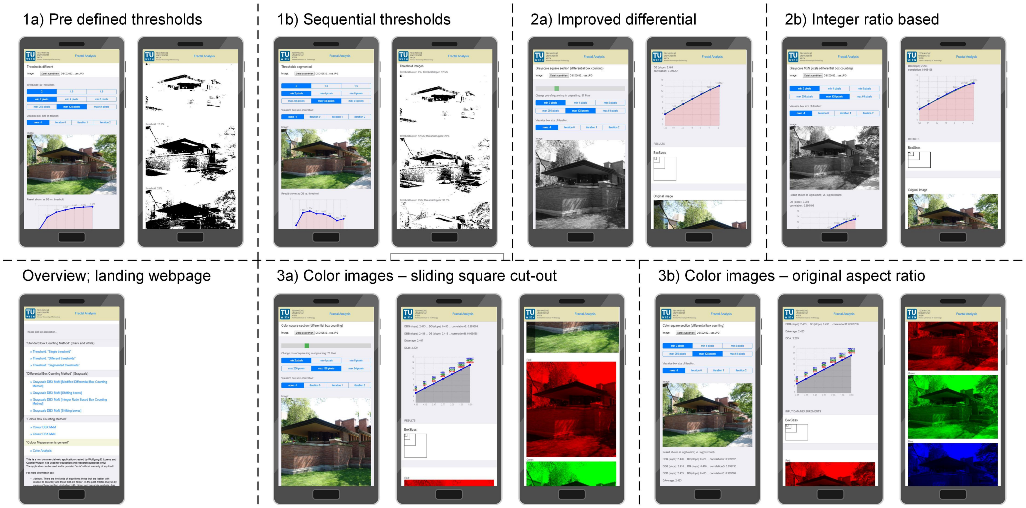

Figure 2
Different measurement methods of FRACAM (from top left to bottom right): 1a+b) “standard box counting algorithm” [1a) pre-defined thresholds and 1b) sequential thresholds]; 2a+b) “differential box counting algorithm” [2a) improved differential box counting method (for square cutouts), with and without shifting the stacked boxes and 2b) integer ratio based, with and without shifting the stacked boxes]; 3a+b) improved differential box counting method for color images [3a) for a sliding trimmed square cutout and 3b) original aspect ratio]; bottom left: the landing webpage.

ject, taking material properties, natural shadowing, and coloring into account. Because of this lack, the authors developed a cell phone application that allows analyzing photographs of facades directly taken with the cell phone camera.

IMPLEMENTATION

Since speed is crucial for the application, all implementations, at first, reduce size. More precisely, the initial photograph is changed into an image of maximal 512 Pixels side length (width or height, respectively). The grid size itself starts with one-fourth of the largest side-length (that is 128 Pixels). Subsequently, the grid-size is halved until the smallest box size of 2 Pixels is reached (reduction factors other than halving are only the exceptions with the following methods). The study of the influence on accuracy caused by the reduction of the image size and reduction factor of grid size will be the subject of future research. What can be said so far is that one clear advantage of resizing to 512 Pixels derives from the algorithm itself: namely, that a reduction of one-half always leads to whole-number side lengths. By that, it is guaranteed that the overall grid size remains the same for all grid scales (this only holds true for those methods that consider square sections).

While the beta version of the application, in general, used a 512 Pixels square-shaped cut-out of the photograph (Wurzer et al. 2017), which influenced the content, the additional implementations cover the whole rectangular input. Moreover, the current version allows for some methods a reduction factors of 1.8 and 1.6.

FRACAM currently consists of the following algorithms: (see Figure 2):

1. the “standard box counting algorithm” operating on black and white images, for a) single conversion threshold, b) pre-defined thresholds and c) sequential thresholds (ranges)
2. the “differential box count algorithm”, operating on grayscale images; this category includes both, the improved differential box counting method applied to a slidable square-shaped cutout of the image and the integer ratio box counting method applied to the original aspect ratio; both cases work either with or without shifting of the stacked boxes (the amount of dis-

placement is predefined and currently equals one Pixel)

3. the "improved box counting method for color images" operating on the 24-bit representation of RGB color images (Nayak et al. 2017); either applied a) to a slidable square-shaped cut-out of the image, or b) to the original aspect ratio

Pre defined thresholds

The first implemented method (Figure 2; 1a) works on the "standard" box counting algorithm. It converts the original photograph into a grayscale image and subsequently in a black and white image using predefined thresholds (currently sequences of 12.5% steps). This leads to a series of results (fractal dimensions) that characterizes the change of roughness while the threshold increases ($T_i; i \in 1 \dots N$). Hence, the tendency characterizes the texture signature (see formula 2; Backes and Bruno 2008).

$$\psi(A) = [D_B \ \ of \ \ T_1, \ \ \dots, D_B \ \ of \ \ T_N] \quad (2)$$

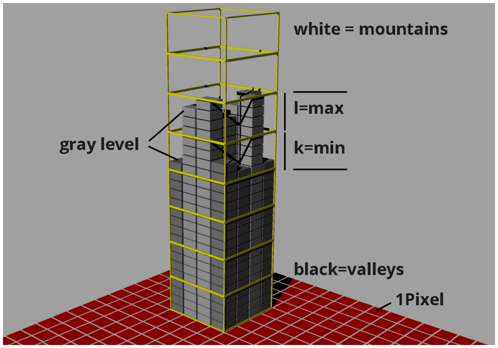

Figure 3
Improved differential box counting method ("boxes" as a yellow grid, grayscale as gray cubes) with (l) being the box number that contains the maximum grayscale level and (k) being the box number that contains the minimum grayscale level.

Sequential thresholds

With the previously described methodology (predefined thresholds), each threshold separates two parts, the "black" foreground and the "white" background. Increasing the threshold means taking brighter and brighter areas of the image into account, which leads to continually higher results. Although a tendency can already be discerned, segmentation may extend readability. Segmentation of grey levels means to use ranges from one threshold to the next instead of considering the entire spectrum in each case (Figure 2; 1b). The clear advantage of segmentation is the improved comparability from one range between thresholds to the next. A horizontal data curve indicates a continuous roughness (similar fractal dimensions) for all segments of grayscale levels. In contrast, outliers with higher values indicate a higher concentration of grayscale levels in a specific segment.

Improved differential box counting method (square cut-outs)

The so-called differential box counting (Sarker et al. 1994) operates directly on grayscale images. This time, the image of MxM Pixels (x, y) is considered as a three-dimensional landscape in which the grayscale level defines the z-coordinate with lighter Pixels forming mountains and darker Pixels defining valleys (see Figure 3). Consequently, the two-dimensional grid is enlarged by stacked boxes (each of which has a size of sxsx$\overline{s}$). While s is given in Pixel, $\overline{s}$ equals the grayscale level and depends on the total number of grayscale levels (G). The following applies:

$$\frac{G}{\overline{s}} = \frac{M}{s} \quad (3)$$

Consequently, with a total number of 256 grayscale levels (G), the largest possible stack height is 256. Unlike the "standard" box counting, which just looks whether or not a box contains a black part, this time, for each box $(i, \ j)$ the difference between the maximum grayscale level (l) and the minimum grayscale level (k) is calculated (see Figure 3). With a ratio

$$\varepsilon = \frac{s}{M} \quad (4)$$

the difference is calculated by

$$n_\varepsilon(i, j) = l - k + 1 \quad (5)$$

Finally, the total number of counted boxes for each grid size covering the full image surface is given as

$$N_\varepsilon = \sum n_\varepsilon(i, j) \quad (6)$$

For covering the grayscale levels completely, the formula of the differential method (5) is slightly modified to (Liu et al. 2014):

$$n_\varepsilon(i,j) = \begin{cases} \left\lceil \frac{l-k+1}{\overline{s}} \right\rceil & \text{if} \quad l \neq k \\ 1 & \text{if} \quad l = k \end{cases} \tag{7}$$

The differential box counting dimension D_{DBC} is, finally, calculated as given in equation (1) and determined by the slope of the regression line in a double logarithmic graph (Figure2; 2a).

With shifting boxes. With the improved method, under-counting is still possible, which may occur at the border of two adjacent stacks of boxes where a sharp grayscale variation exists (black and white variation). Therefore, following Long et al. (2013) and Liu et al. (2014), a further improvement is achieved by shifting the stacks of boxes in the $x-y$ plane. To reverse the direction of shifting at the border prevents boxes from running out of the image (i.e. the program recognizes whether the stacked boxes are located at the edge of the picture and reverses the direction consequently; from this follows, that the shifted stacked boxes overlap with the previous ones). The original position of the stacks leads to a resulting $n_\varepsilon(\text{original})$. This result is then compared with $n_\varepsilon(\text{shifted})$ where the following applies

$$n_\varepsilon = \max\left(n_\varepsilon(\text{original}, n_\varepsilon(\text{shifted})\right) \tag{8}$$

The total number of boxes N_ε is calculated according to (6) and the fractal dimension is determined by the slope of the regression line of the double logarithmic curve (log (N_ε) versus log $(1/\varepsilon)$) with $\varepsilon = s/M$.

Integer ratio based box counting method

The improved differential box counting method works well with square-shaped images. This especially holds for images of 512x512 pixels (x- and y-direction), since in this case, 256 gray levels (z-direction) is a fraction of the side length. Consequently, the ratio is the same for all boxes with $\varepsilon = s/M$ (where (3) applies).

With rectangular grids, however, four cases have to be distinguished (Long et al. 2013); such a distinction depends on dividing the whole image MxN into boxes of mxn pixels, where the following applies: a) $M = m$xε and $N = n$xε for an even partitioning in x- and y-direction; b) $M > m$xε for an uneven distribution in x-direction; c) $N > n$xε for an uneven distribution in y-direction; d) $M > m$xε and $N > n$xε for an uneven distribution in x- and y-direction. The number of boxes is then calculated by

$$n_\varepsilon(i,j) = \text{ceil}\left(\left(\frac{l-k+1}{p}\right)\cdot\left(\frac{S_{i,j}}{m\cdot n}\right)\right) \tag{9}$$

with $S_{i,j}$ being the area of a single box and p the height of the box ($p = G/\varepsilon$). According to cases two to four from before, $S_{i,j}$ can differ from mxn ($S_{i,j} = m$xn only holds true for the entire grid with an even partitioning). The total number N_ε for each grid size is calculated as given in formula (6). Finally, the slope of the regression line in a double logarithmic graph determines the fractal dimension ($D_M BC$). Similar to the improved box counting method, one implementation of the algorithm works with and one without shifting the stacked boxes.

Improved differential box counting method for color images

The improved box counting method for color images is applied directly to a 24-bit representation of an RGB color image (Nayak et al. 2017). With this method, each of the three color components (R, G, B) is analyzed in separation (after a separate conversion to grayscales). The method used is the improved differential box counting method, resulting in $D_{DBC(R)}$, $D_{DBC(G)}$, $D_{DBC(B)}$. After subtracting the corresponding smoothness ($D_R = D_{DBC(R)} - 2$, $D_G = D_{DBC(G)} - 2$, $D_B = D_{DBC(B)} - 2$) all components are added to smoothness again:

$$D_Col = D_R + D_G + D_B + 2 \tag{10}$$

One implementation of the algorithm works with a square-shaped cut-off of the picture (MxM); while the other one works with the original aspect ratio (MxN; similar to the integer ratio based box counting method). From the results so far, it looks as if a smaller part of the picture already allows conclusions on the overall picture (different details of the brick wall: $D_{\text{average(2-128Pixels)}} = 2.44 - 2.49$; overall view: $D_{\text{average(2-128Pixels)}} = 2.42$; compare Figure 4). That is not surprising since each picture shows a main motif (the building); i.e. there are only little differences in the characteristics to be expected (apart from building versus surrounding).

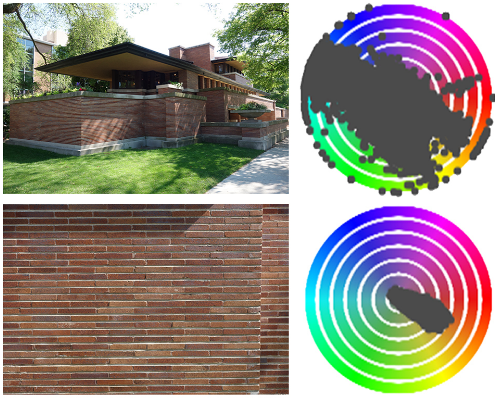

Figure 4
Color distribution of Robie House (overview and detail).

ANALYSES OF CASE STUDIES

The authors have carefully tested all implemented algorithms (including those based on Backes and Bruno 2008, Gao et al. 2016, Sankar et al. 2010) with regards to real-time application and correctness, using specially prepared color images and cell phone photos of iconic buildings (e.g. Robie House by Frank Lloyd Wright). The main goal was twofold: a) to determine influences caused by photography (e.g. different lightning and coloring as a consequence of daytime, but also influences due to the portion of green plants or blue sky) and b) to determine influences caused by the methods. Altogether 26 images were examined: seven photographs of Robie House (overviews and details), six color images (including a pure red, green and blue image, but also a tricolor image), three photographs of skyscrapers (including the 860-880 Lake Shore Drive and the Federal Centre Complex by Mies van der Rohe), two photographs of a sculpture (the “flamingo” by Alexander Calder) and eight photographs of other buildings.

Camera Settings

When analyzing the influences of photography on measurement methods, it is important to take camera settings into account. These settings include exposure time, ISO, focal length [mm], and lightness. Admittedly, at this stage of research, no clear correlations can be made since the number of photographs examined is too small. However, the authors want to point out the importance of this aspect and present first results.

Color analysis

Besides camera settings, this paper also examines the effects of colors (present in a picture) on the resulting fractal dimension. This especially holds for the improved differential box counting method for color images. Color analysis includes the average color, the average color total and brightness [4]; the prominent colors called vibrant, muted, dark vibrant, dark muted and light vibrant [5]; the dominant color and the palette of colors each with aggregation [6]; the ten most prominent (major) colors with tolerance, the number of predefined color ranges (see Figure 4), saturation and lightness. Some of the color analysis methods listed are based on freely available internet sources (getAverageColourAsRGB.js [4], vibrant.js [5], color-thief.js [6]). All color analysis algorithms are also part of the web application by the authors, freely available on the internet [7].

DISCUSSION

The authors are aware that all fractal measurement methods are subject to certain influence factors (Fourtan-Pour et al. 1999). This leads, on the one hand, to the necessity of post-processing with statistical methods (Lorenz 2012). On the other hand, the authors believe that it is necessary to use various methods and to define a comparison methodology - all the more so because each method requires a specific preparation of what is measured and thus measures different content (line drawings, grayscale, or color images). Differences occur, for example, because CAAD plans and photographs offer a different kind of detail richness and consequently either include main features of a design intention or show the real built object (in its surrounding). But differences also occur because of the chosen view, either as frontal elevation or as a perspective view. Consequently, it is necessary to store the image in a database together with the results (D) and with data about the time of year, daytime, light conditions, and position. However, this will be the subject of future investigations.

Although only a few selected case studies are analyzed so far, trends can already be recognized.

Results

As expected, with the simple threshold method, the fractal dimension generally increases the higher the threshold (with only a few exceptions). If the photograph already shows a low contrast with similar hues, the curve is flattest. In contrast, sequential thresholds lead to a more individual course, mostly with higher values in the middle areas. These peaks seem to apply especially to details of exposed brick walls (see Figure 5). In contrast, overviews tend to have a flatter course, with Johnson Wax Headquarters having even a valley at segment 37.5%-50%.

Concerning the differential box counting method, all four mutations show similar tendencies. That means the values rise and fall to a similar extent. The relative differences (between all four variations) are within a range of 5 to 10 percent. A minimum box size of 8 pixels leads to the smallest differences (the deviation lies between 1 and 5 percent). The lowest values occur with images with original aspect ratio MxN without shifting the stacks of boxes; the highest values occur with square-shaped cut-offs (MxM) with shifting. It can also be stated that a minimum box size of 2 pixels leads to the lowest results, while a minimum box size of 8 pixels leads to the highest results. The differences are within the range of 6 to 9 percent and 4 to 8 percent, respectively (within the same method). At the same time, the available data also shows that shifting the stacks of boxes leads to higher values than without shifting (with the difference being below 1 percent). That means that this option does not have a big effect.

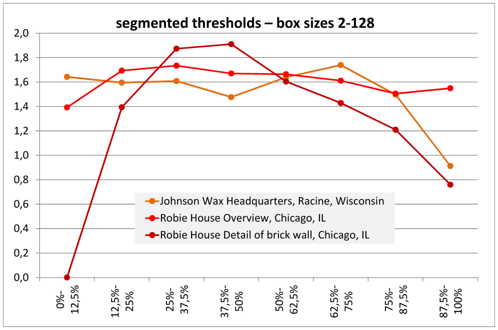

Figure 5
Results of the sequential threshold method for Robie House (overview and detail of brick wall) and Johnson Wax Headquarters.

Like grayscale analysis, color methods show similar tendencies for all mutations, i.e. values rise and fall to a similar extent (that holds true for the red/-green/blue parts as well as average values). Again, a minimum box size of 2 pixels leads to lower results compared to 8 pixels. The differences between the red, green, and blue parts are small within one and the same mutation and are within a range between 1 and 5 percent; this time, the differences between the analysis of square-shaped cut-offs (MxM) and the picture with the original aspect ratio (MxN) are even smaller than with other methods. There are slight differences for the average as well as for the color dimensions if the minimum box size is 2 pixels (see Figure 6, with D_{col} being higher than the average and the color dimensions); these differences increase slightly for a minimum box size of 8 pixels.

The comparison of Lake Shore Drive with Robie House serves as an example of data evaluation. The first example belongs more likely to Euclidean geometry, since it exhibits smoothness of its glass facade with a strong linearity and an orthogonal structure. In contrast, Robie House has some fractal characteristics, such as self-similarity and roughness across different scales (Lorenz 2004). Surprisingly, both examples show only slight differences concerning the different measurement methods. The reason lies in the fact that the glass façade reflects the surrounding buildings; this dissolves the smoothness, the uniformity. Larger differences only arise with the sequential thresholds, where the results of details of the Robie House follow a more curved characteristic (see Figure 5).

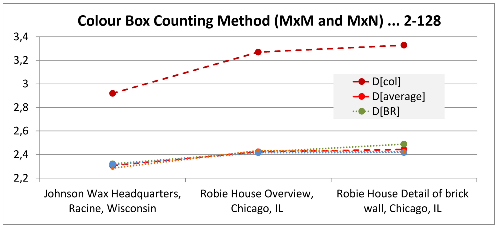

Figure 6
Results of the improved differential box counting method for color images.

Correlation

The correlation between fractal dimension and color analysis methods provide insight into their relationship. In more detail, the correlation coefficient determines whether there is a relationship between the two properties. No correlation is indicated by a coefficient equal to 0, while a perfect correlation exists when the coefficient equals +1 or -1. There is a positive correlation if both series of numbers tend in the same direction (+1); while a negative correlation is indicated by the opposite movement (-1).

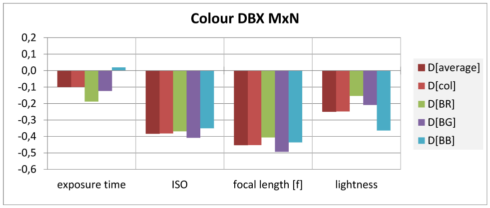

Figure 7
Correlation between improved differential box counting method for color images and exposure time, ISO, focal length, and lightness.

So far, no correlation can be found between any of the before mentioned methods, and the average color of image [01], the number of predefined color ranges, the saturation or the lightness. The same holds true for the relation between camera settings and fractal dimension. The latter has already been shown by C. Horvath und P. Stummer (2020) using 115 pictures (including Robie House and other buildings located in Chicago). In their paper, they showed that for grayscale measurements the f-number, the presence of reflecting elements, and the year of construction tend to have a weak negative correlation, while exposure time [1/sec] and focal length [mm] have a weak positive correlation. As part of this paper, the authors examined 20 photos, which show a weak positive correlation for exposure time, ISO, focal length, and lightness; this holds true for both, the integer ratio based box counting method and the improved differential box counting method for color images (see Figure 7). It can be concluded that apparently neither the color of an image nor the camera setting significantly influence the results, but only what is shown (the building and its surrounding).

CONCLUSION

(Architectural) Quality is a relative term, which means it follows specific criteria. The authors argue that one of these criteria includes characteristics of fractal geometry that concern the description of perception (in more detail, the statistical self-similarity). Various methods for measuring the fractal dimension - as a linkage between different scales (characteristics of self-similarity) - are presented in this paper, including the "standard box counting algorithm", the "differential box count algorithm" and the "improved box counting method for color images". The implementation as a web application, called FRACAM, makes usage easy and is ideal for measuring architectural photos. It could be shown that neither camera settings nor the color of the image seem to influence the result, but only the content. A connection to style or type of building cannot be given at the moment since sufficient data is still missing; this will be investigated in future work. Future work will also deal with the possibility of placing the result in relation to other examples. Such a comparative measurement means to consider facades as a member of a set with classification and ranking in comparison with other buildings (including iconic buildings). This feature will be made possible by storing all information (including the photographs, the time of year, daytime, light conditions, position, and results) in a database.

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