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# A comparison between mixed-integer linear programming and dynamic programming with state prediction as novelty for solving unit commitment

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ABSTRACT

Recently, the increasing prevalence of renewable energies has faced the challenge of operating power supply systems to efficiently plan electricity generation on a daily basis, since renewable energies are generated intermittently and the decisions of the individual generation units are discrete. The Unit Commitment (UC) problem, which determines the dispatch of generation units, is one of the critical problems in the operation of power supply systems. A long list of formulation proposals have been made that claim to solve this problem. For this purpose, two established approaches, mixed-integer linear programming (MILP) and backward dynamic programming (DP), are used as basis for a deterministic single-generator unit with general convex cost function in this paper. The DP algorithm is enhanced by a so-called state prediction, which reduces the time to find the optimal solution. The proposed formulation is tested empirically on the basis of existing formulations at long-term profit based UC instance derived from real data. Finally, the calculation results show that the derived approach significantly shortens the computation time, which confirms the effectiveness of state prediction. The comparison of the approaches shows that the DP algorithm with state prediction delivers a satisfying solution in significantly less time than DP and MILP. Furthermore, the given linearity of the dependence of the computation time on number of steps is a superior advantage of the DP strategy. This superiority becomes even more evident when the planning horizon extends over a longer period of time.

## 1. Introduction

The short-term to long-term scheduling of power plant dispatches has become more and more difficult due to volatile market prices for fuel procurement, electricity sales, take-or-pay quantities and other aspects. Furthermore, the number of generating units is also increasing, thus raising the degree of complexity of the whole system. Fossil fired power plants which are mostly meant to run throughout the year are more often switched on and off. Their full load hours are decreasing which leads to higher costs. In the meantime, the electricity surplus produced by renewable sources is increasing and if no new controllable power plants are built, the back-up capacity is decreasing, since old fossil-fired power plants are being shut down.

There are several different solution methods for optimisation tasks, which are treated as unit commitment problem. The unit commitment task involves planning the on/off status and the actual power output of the thermal units to cover the forecasted demand over a finite time horizon. The resulting schedule should minimize system production costs over a period of time while meeting the load requirements as well as the physical and operational constraints of the individual unit. Because electricity suppliers can save a significant amount of money in production costs annually by applying an improved UC schedule. Therefore, UC is an important optimization task in the daily operational planning of modern power supply systems [1]. In the last decade, MILP (mixed-integer linear programming) has widely replaced DP (dynamic programming) due to increasing computing power. For example [2], provides an accurate and computationally efficient MILP formulation to address the UC problem with a single unit and maximize overall profit. The duration of the solution vastly depends on the size and complexity of the problem. For many utilities, solving the unit commitment can take several hours, which is considered as critical issue. For this reason, they sometimes have to accept cutbacks in terms of time resolution and a shorter planning horizon in order to reduce the computation time.

In this paper, a classic MILP method is compared with a DP algorithm by solving profit based single unit commitment. Furthermore, the DP approach is enhanced by a state prediction, which supports the

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**Nomenclature**

*Indices and Sets*

| | |
|---|---|
| $i \in I$ | generating units, running from 1 to $I$ |
| $s \in S$ | start-up categories for generator $i$, from hottest (1) to coldest ($S_i$) |
| $t \in T$ | 15 min periods, running from 1 to $T$ |

*Parameters*

| | |
|---|---|
| $\Delta T$ | time grid factor for planning horizon (15 min base) |
| $T_{i,down}$ | time down after another startup is allowed for generator $i$ (h) |
| $T_{i,run}$ | minimal up time after finished startup for generator $i$ (h) |
| $\Delta P_{i,max}$ | maximum power gradient (ramp-up, ramp-down during online) (MW/15 min) |
| $\alpha_i$ | power offset of inverse production function for generator $i$ ($MW_{th}$) |
| $\beta_i$ | linear part of inverse production function for generator $i$ ($MW_{th}$/MW) |
| $\gamma_i$ | quadratic part of inverse production function for generator $i$ ($MW_{th}/MW^2$) |
| $q_{i,0}$ | power offset of approximated production function for generator $i$ ($MW_{th}$) |
| $q_{i,1}$ | linear part of approximated production function for generator $i$ ($MW_{th}$/MW) |
| $f_{CO_2}$ | $CO_2$ emission factor (t/$MW_{th}$) |
| $T^s_{i,down}$ | minimum required time for $s$ start-up start since last shut-down for generator $i$ (h) |
| $T^s_{i,up}$ | duration of $s$ start-up for generator $i$ (h) |
| $T^{shut}_{i,down}$ | duration of shut-down for generator $i$ (h) |
| $S_{i,s}$ | $s$ start-up costs for generator $i$ (EUR) |
| $P_{i,O\&M}$ | operation and maintenance costs for generator $i$ (EUR/h) |

*Variables*

| | |
|---|---|
| $p^{market}_t$ | market price at time $t$ (EUR/MWh) |
| $P_{i,t}$ | scheduled power for unit $i$ at time $t$ (MW), $\geqslant 0$ |
| $P^{th}_{i,t}$ | scheduled thermal power for unit $i$ at time $t$ (MW), $\geqslant 0$ |
| $P_{i,t,min/max}$ | minimum/maximum power output for generator $i$ at time $t$ (MW), $\geqslant 0$ |
| $p^{fuel}_{i,t}$ | fuel price for unit $i$ at time $t$ (EUR/MWh), $\geqslant 0$ |
| $p^{markup}_{i,t}$ | mark-up costs for unit $i$ at time $t$ (EUR/h), $\geqslant 0$ |
| $p^{o\&m}_{i,t}$ | operation costs for unit $i$ at time $t$ (EUR/MWh), $\geqslant 0$ |
| $u_{i,t}$ | commitment status for unit $i$ at time $t$, $\in 0, 1$ |
| $\tilde{u}_{i,t}$ | start-up/shut-down status for unit $i$ at time $t$, $\in 0, 1$ |
| $z_{i,t,s}$ | type of start $s$ for unit $i$ at time $t$, $\in 0, 1$ |
| $y_{i,t}$ | start-up status for unit $i$ at time $t$, $\in 0, 1$ |
| $w_{i,t}$ | shut-down status for unit $i$ at time $t$, $\in 0, 1$ |
| $SC_i(z_{i,t,s})$ | starting costs for unit $i$ depending on start type $z_{i,t,s}$ (EUR), $\geqslant 0$ |
| $F_i(q_{i,t})$ | fuel costs for unit $i$ depending on the production function $q_{i,t}$ (EUR), $\geqslant 0$ |
| $M_{i,t}$ | maintenance costs for unit $i$ at time $t$ (EUR), $\geqslant 0$ |
| $A_{i,t}$ | additional costs for unit $i$ at time $t$ (EUR), $\geqslant 0$ |
| $C_{i,t}$ | sum of costs for unit $i$ at time $t$ (EUR) |

search for the optimum and helps to reduce its computation time. Memoization, which is a well-known technique for DP approaches, is used to reduce the memory requirement. The superiority of the DP approach with state prediction regarding computation time is shown by comparing them with a classic DP algorithm. Its results are verified by comparing it to the results of an already established DP algorithm which is currently used from the utility for the unit's dispatch. A combined-cycle gas turbine (CCGT) unit serves as the dispatchable object, which is optimised against the market price with the goal to maximise revenue (profit based single UC).

The structure of the paper is as follows: Section 2 presents the state of the art of power plant dispatching and upcoming challenges with a focus on unit commitment. Section 3 outlines the major cornerstones, applied methods and syntax of the two approaches used for the analyses. Section 4 presents the results of the mixed-integer linear programming and the backward dynamic programming approach. A sensitivity analysis with a focus on variations and other aspects is presented in Section 5. In Section 6, the results are compared, discussed and the validation of the algorithm is performed. Finally, a conclusion of the outcome is presented in Section 7.

## 2. State of the art

In the literature, there are several approaches for solving the unit commitment problem. A long list of formulations that claim to solve this problem more efficiently has been proposed. The interplay of shortened forecast horizons and rapid shifts in net demand, coupled with the relative inflexibility of traditional power generation, poses a significant challenge. Therefore, a considerable effort is put into this science [3–5]. Starting up a plant is costly, especially when the wind picks up again and it turns out that starting up was unnecessary. Similarly, shutting down a unit is risky, as the unit may not be able to operate for some time due to its minimal downtime, should the net load suddenly increase. Due to the lack of flexibility, the answer to this problem has been an increase in part load in many systems. Several scientific contributions focus on solving the unit commitment problem and other factors that pose a great challenge. The issues regarding unit commitment as well as a selection of the most important influences on this paper are mentioned in the following:

### 2.1. Unit commitment solving approaches

Unit commitment has more than just one goal [6,7]. The objective is to calculate a generator schedule which is required for.

1. minimization of the total cost
2. supply the total demand
3. meet the technical and security constraints.

In the short term, this is primarily achieved through successful control of the system [8]. A noteable tool for minimising costs purpose is the merit order approach [9]. Another factor to bear in mind is the accuracy of long- and short term forecasts, as unnecessary start-ups will inevitably translate into higher operating and maintenance costs [10,11]. The objective has to be achieved under several operating constraints, which have to be satisfied. The constraints reduce the unit's freedom of starting up and shutting down [12,4].

There exist some approaches for solving unit commitment problems, all of which have their advantages and disadvantages [13]. A very basic approach is the priority list, which is the simplest and fastest one but delivers insufficient solutions. Dynamic Programming approaches can be fast depending on the problem according to [14] but suffer from high computation times when it comes to the so-called curse of dimensionality [15].The Lagrangian relaxation is a faster approach but suffers from numerical convergence [16,17]. The resulting duality gap is a common challenge [18]. Examples and further examinations of mixed-integer linear programming approaches are shown by [19,20]. MILP is an easy-to-implement solution but struggles when the number of units

increases [21]. This solution technique requires large amount of memory and suffers from high computation time. Another approach is the branch-and-bound method [22], which uses linear functions representing start-up and fuel costs. The disadvantage of this method is the exponential growth of the computation time for large systems [23,5]. In this paper, the dynamic programming and the mixed integer linear programming approach are modelled for a basic power system due to their frequent use in real systems.

In terms of MIP formulations, two metrics have been defined in recent research to compare performance of different formulations [24,25]:

- tightness - determines the search space that needs to be looped through in order to find the optimal solution. The tightness can be calculated using the distance between the LP and MIP solutions (integrality gap).
- compactness - refers to the problem's size, which depends on the number of constraints, variables, and non-zero elements. A formulation is more compact than others if its size is smaller.

Nevertheless, tightness and compactness can not be necessarily translated into CPU time. For the purpose of this examination, detailed ramping constraints are considered to obtain a more realistic definition of the UC problem, as provided in [26,27]. In this research, the authors remark that the state transition formulation performs best, especially for long-horizon instances, which serves as inspiration for this formulation. Moreover, in reference [28], the authors do not include start-up and shut-down trajectories in their proposal, which is examined in the current research.

### 2.2. Factors with high influence on unit commitment

- Despite the significant increase of renewable energy sources (RES) in generation contribution, most power supply systems are still mainly based on thermal and hydropower plants. Indeed, these are necessary to cope with the uncertainty in production performance, typical of most RES units, which leads to highly complex, uncertain stochastic UC variants [29–31]. Therefore, thermal units still remain at the core of basically every UC model.
- The combined-cycle unit (CCGT) is very popular because of its high efficiency, fast response, shorter installation time, and environmental sustainability. For instance, it enables the combined-cycle unit to hedge against the uncertainty caused mainly by RES [32]. Most combined cycle power plants were designed and optimised for stationary base load operation. However, they were not designed for frequent starting and stopping procedures. As a result, the various components of a steam plant are always exposed to different thermal loads during the start-up process [33]. Over time, this leads to a variety of problems, including reduced equipment life, increased maintenance, increased downtime, and reduced efficiency [34]. Due to the implied cost of start-ups, one goal of dispatching is to minimise the number of start-ups required during the day. This problem is exacerbated by the shortened forecast horizon, which itself is a direct consequence of the increased volatility. When using a CCGT unit in a traditional unit commitment, problems could occur in terms of accurate modeling of the unit components which are well addressed in [32].
- Due to the increasing number of participants in the supply market, the level of volatility has also increased [35]. Therefore, finding a solution to the unit commitment problem becomes increasingly challenging [4]. The challenge is even greater in wind and solar power plants, since the power output of these two sources does not correlate most of the time and can create situations that are difficult to handle [36,37]. Taking into account the intermittent behaviour of increasing penetration of renewable generation and generation amount dependency among time periods, a stochastic approach of the deterministic UC is formulated in [31]. In this paper, a deterministic profit based single generator UC is examined with focus on time-dependent start-up and shut-down trajectories as well as time-dependent start-up costs. Intermittent generation has a magnifying effect on load changes for saleble production [38]. These rapid shifts result in a strongly cyclical operation, e.g., start-ups, shutdowns and ramping as well as part-load operation of ready-to-dispatch units with obvious cost implications. The implied long-term costs of start-ups are becoming more prevalent in the industry. Less recognised is the fact that ramps, whether up or down, are associated with long-term costs, in contrast to start-ups.
- Some decisions, such as ramping up a power plant, need to be known in advance and within a specific time frame [39]. A noteworthy consequence of increasing the degree of complexity are uncertainties in the power system. Some uncertainties aggregated with demand and generator outages can be well forecasted [40,41]. The challenge of prediction is compounded by the fact that, due to basic mathematics, increasing errors in prediction are an inevitable by-product of increasing variability in production. Forecasting difficulties increase the number of unnecessary start-ups and shutdowns. They also increase the requirements for the ramp, which impacts the costs [42]. Consequently, decisions are made based on forecasts that are less reliable than before.
- The increasing use of partial load decreases system level efficiency and consequently, increases the cost of fuel per unit of power produced. The partial load increases not only the rising fuel costs but also the emissions per unit of power generated. Moreover, modern emission control technologies do not operate optimally under unstable conditions. In other words, the emissions are always higher at start-up, shut-down and ramps than at stable operation under full load [43].
- The daily load fluctuation and in particular the difference between the daily peak load and the load during the night increases rapidly in many power supply systems [44]. The prime example of this development is Japan, where the daily peak load, which usually occurs in the evening, is twice as high as the lowest load point in the middle of the night [45]. Increasing daily load fluctuations have led to more and more generating units being started each morning and shutting down every night, resulting in higher operating and maintenance costs. Due to the increasing volatility of the net load demand, the dispatchable power generation units has to increase significantly, whereby the use of these components increases [46].
- The main research on UC focuses on short-term planning horizons (e.g. Day-Ahead to one week). For this scheduling, production control activities are done, where unexpected outages or shifts in demand are included in real-time. In contrast to long-term problems which aim at production planning and where full-load hours, operating costs and fuel demand are relevant. Based thereon, a long-term planning horizon (one year) is considered in this work to provide strategic information for utilities. Those companies then have the ability to estimate fuel demands, maintenance activities and determine the economic performance of the generation.

### 2.3. Progress beyond the state of the art

Above all, the combination of the previously mentioned aspects makes the already demanding task of solving the unit commitment problem in a short amount of time extremely difficult. As a result, system operators often have to use sub-optimal solutions to meet the time limit, i.e. with rather big optimality gaps [47]. An alternative approach for reducing run times is to develop stronger formulations for UC. Traditional UC models assume that units have a start-up/shut-down capacity such that the unit can be started up or shut down in one period. However, there are thermal units that need more than one period to either start-up or shut-down depending on the type of start-up, i.e., hot, warm, or cold. Therefore, UC models ignore the intrinsic

start-up and shut-down power trajectories of thermal units. These trajectories ensure the ramping requirements of the unit during the start-up and shut-down. Moreover, the authors in [27] discuss how ignoring these power trajectories can significantly change commitment decisions, which leads to an increase in operating costs. Despite this, start-up/shut-down trajectories are commonly ignored because the resulting model is more complex and time-consuming [13]. In the last few years, ramping constraints were complexified in [27] to gain more realistic formulations [21]. The aim of this paper is to create an approach for long-term optimisation with as few simplifications as necessary without greatly increasing the time required for calculation. In order to achieve this goal, a backward dynamic programming approach motivated by [48] is used as a basis which contains a polynomial number of variables and constraints and is extended with an algorithm for state prediction. Currently, the most extensively used approach is the priority list technique [49]. In this work a faster dynamic programming algorithm that run in less time to solve the deterministic single UC problem is developed. Most of this work focuses on tightening the formulation of a single generator's dispatch when start-up and shut-down trajectories are used with time-dependent start-up costs. The validity of the new approach is proven by comparing it with an implemented tight and compact MILP approach using [25] as basis. Furthermore, it is shown that the DP approach with the state prediction decreases the computation time, proven by comparing it with a classic DP algorithm which has been recognized to be computationally efficient in practice.

## 3. Methodology

This section is devoted to the description of the mathematical model. This includes general properties and parameters that describe the model and which conditions or constraints it must fulfil. Two approaches, mixed-integer linear programming (MILP) and backward dynamic programming (DP), are used to solve the unit commitment problem. The focus lies on the mathematical formulation of these approaches. The test case is a particular CCGT plant with several units from a local utility in Austria. They are modeled as one simple generator in a profit based UC. The plant is a so-called price taker, which means that its operating condition does not affect the price of electricity in contrast to that of a market maker, which is of fundamental importance, since the market price can be considered as independent from the dispatched power. The data for the electricity price is sourced from the EXAA (Energy Exchange Austria). The time step interval corresponds to a resolution of 15 min but is adjustable by $\Delta T$ and further minor adjustments in the problem formulation. A large number of time steps in combination with a long planning horizon increases the complexity of the problem tremendously, which has a direct impact on the computation time. For this particular test case a planning horizon of one year and a time grid of 15 min is chosen to achieve a larger gap in the computation times for the different approaches. Another aspect which is condidered are detailed ramping trajectories which are typical in such a fine time grid.

The efficiency of the power plant unit is not constant and depends on the electricity power output. The relationship between electrical demand and thermal fuel power is modelled by a quadratic inverse production function, which is shown in Eq. (1). It is almost piecewise linear and can be approximated by Taylor Series, which leads to Eq. (2). Such approximations are essential for solving a unit commitment problem without significantly disorting the results. The quadratic inverse production function is only valid, when the unit is in online status. During a ramping process (start-up or shut-down) the power output corresponds to a thermal power level that is not in accordance with the production function. These situations have to be treated separately.

$$P_{i,t}^{th} = \alpha + \beta \cdot P_{i,t} + \gamma \cdot P_{i,t}^2 \tag{1}$$

$$P_{i,t}^{th} = q_{i,0} + q_{i,1} \cdot P_{i,t} \tag{2}$$

### 3.1. Mixed integer linear programming (MILP)

Mixed-integer linear programming is a practical modelling approach for problems for which there exist no explicit algorithms. In real applications, MILP is widely used and successfully applied in unit commitment problems.

The overall total cost function $TC$ for several thermal units $i = 1...I$ in a particular time interval $t = 1...T$ is shown in Eq. (3) with $u_{i,t} = 1$ if unit $i$ is on at time step $t$ and 0 if unit $i$ is off, or in the process of start-up or shut-down at time step $t$.

$$TC(u, \tilde{u}, q) = \sum_{i=1}^{I} \sum_{t=1}^{T} C_{i,t}(u_{i,t}, \tilde{u}_{i,t}, q_{i,t}) \tag{3}$$

The cost function that is shown in Eq. (4) consists of fuel costs, start-up costs, operation and maintenance costs and additional costs, where the fuel costs $F_i(q_i)$ are modelled as an approximated inverse production function of $q_i$, and the start-up costs $SC_{i,t}$ depend on the type of start-up $z_{i,t,s}$ (hot, warm, cold) as well as the maintenance costs $M_{i,t}$ out of $p_{i,t}^{o\&m}$ and additional costs $A_{i,t}$ that depend on $f_{CO_2}$ and $p_{i,t}^{markup}$.

$$\begin{aligned} C_{i,t}(u_i, \tilde{u}_i, q_{i,t}) = {} & (u_{i,t} + \tilde{u}_{i,t}) F_i(q_{i,t}) + u_{i,t}(1 - u_{i,t-1}) SC_{i,t}(z_{i,t,s}) \\ & + (u_{i,t} + \tilde{u}_{i,t}) M_{i,t} + (u_{i,t} + \tilde{u}_{i,t}) A_{i,t} \end{aligned} \tag{4}$$

In the realisation of the model with MatLab, the solver Gurobi Version 7.5 is used. It uses a combination of branch-and-bound, presolving, and cut-plane methods and heuristics. The maximisation of the revenue as a target function is shown by Eq. (5). The objective function consists of the profit generated by marketing the electricity price mines the purchase of fuel, additional costs, operation & maintenance costs, and start-up costs.

$$\begin{aligned} \prod = {} & \max_{P_{i,t}} \sum_{i=1}^{I} [\sum_{t=1}^{T} (p_t^{market} \cdot P_{i,t} \cdot \Delta T) \\ & - [p_{i,t}^{fuel} \cdot (q_{i,0} \cdot u_{i,t} + q_{i,1} \cdot P_{i,t}) + (p_{i,t}^{markup} \cdot P_{i,t}) + (p_{i,t}^{o\&m} \cdot u_{i,t})] \cdot \Delta T \\ & - \sum_{s=1}^{S} (SC_{i,s} \cdot z_{i,t,s})] \end{aligned} \tag{5}$$

The objective function is subject to several constraints. In the course of this examination minimum up and down times, time-dependent unit generation capabilities, exact ramping up and down rate constraints with respective trajectories including time-dependent start-up costs and must-run constraints are concerned. They are described in the following.

The power plant unit has to maintain certain minimum up and down times. After a start-up the unit has to stay at least $T_{run}$ on, as represented by Eq. (6). When the unit has been shut down it has to stay off for $T_{down}$, as shown by Eq. (7).

$$u_{i,t...t+T_{run}} \geqslant u_{i,t} - u_{i,t-1} \tag{6}$$

$$u_{i,t...t+T_{down}} \leqslant (1 - u_{i,t-1} + u_{i,t}) \tag{7}$$

The start-up action $z_{i,t,s}$ and the shut-down action $w_{i,t}$ are determined by the unit commitment status in the previous time period $u_{i,t-1}$ and the current time period $u_{i,t}$, as shown in Eq. (8). This constraint ensures that a unit cannot start-up and shut-down simultaneously.

$$u_{i,t} - u_{i,t-1} = y_{i,t} - w_{i,t} \tag{8}$$

The binary variable $y_{i,t}$ signals a start-up. In addition to this, $z_{i,t,s}$ preserves the information regarding the type of the start-up. Only one binary variable of $z_{i,t,s}$ is active at time step $t$, which is represented by Eq. (9). The constraints for the start-up types $z_{i,t,s}$ are shown by Eq. (10).

$$\sum_{s=1}^{S} z_{i,t,s} = y_{i,t} \tag{9}$$

$$z_{i,t,s} \leqslant \sum_{j=T^s_{i,down}}^{T^s_{i,down}-1} w_{i,t-j} \tag{10}$$

The scheduled power $P_{i,t}$ can vary between $P_{min}$ and $P_{max}$ when the start-up process has been finished. Otherwise, the power is zero, or the unit is in a ramping process. The unit can be considered online if the power output is greater than $P_{min}$, as shown in Eq. (11). However, it has to complete a start-up process first to reach minimum power. This equation is further constrained by ramping processes, which are discussed in the following subsection.

$$P_{i,min} u_{i,t} \leqslant P_{i,t} \leqslant P_{i,max} u_{i,t} \tag{11}$$

There are two ways of achieving proper ramping. The default approach is addressed by Eq. (12), where $\Delta P_{max,i}$ is the maximum power gradient of unit $i$. This behaviour is only used when the power plant is in online status, the start-up launch has finished, and no shut-down is processed.

$$-\Delta P_{i,max} \leqslant P_{i,t} - P_{i,t-1} \leqslant \Delta P_{i,max} \tag{12}$$

The ramping-up behaviour depends on the previously resulting down-time. Fig. 1 illustrates the full operating of a start-up and shut-down process.

The distinction of $u_{i,t}$ and $\tilde{u}_{i,t}$ is fundamental for the operation behaviour. Particularly, the ramping up and down processes of the unit $i$ are displayed by $\tilde{u}_{i,t}$ and shown in Eq. (13).

$$\tilde{u}_{i,t} = \begin{cases} 0, & \text{if unit } i \text{ is on at time } t \\ 1, & \text{if unit } i \text{ is in startup or shutdown process} \end{cases} \tag{13}$$

In most cases, a particular ramping situation (hot/warm/cold) is required when the unit is launched, in which ramping rate is a changeable power value. It is also affected by the previous time period of ramping and commitment status, as shown in Fig. 2.

The default ramp-up and ramp-down rate is not suitable under this condition. For the case of the three different starting types, which differ in their power values as well as launching times while sharing one down type, the model in Eq. (12) is enhanced. To fullfil this constraint, it has to be ascertained what kind of start-up $s$ is taking place, which is determined by $z_{i,t,s}$. In accordance with these binary variables, the power limitations $P_{i,t,min}$ and $P_{i,t,max}$ are adapted by $P_{i,t,s}$ and corresponds to the respective power value. Therefore, the lower power bound is set for ramping up. The power bound $P_{i,t,s}$ is calculated from the given ramp curve. Similarly, the ramping down process is modelled by adapting the upper bound $P_{i,t,max}$. During the shut-down process, $P_{i,t,down}$ is unequal to 0 and $w_{i,t}$ is set to 1. Therefore, the normal $P_{i,t,max}$ is set to zero, and the corresponding shut-down power value is used as constraint instead. The complete ramping behaviour is shown in Eq. (14).

$$(u_{i,t} \cdot P_{i,t,min}) + \tilde{u}_{i,t} \cdot P_{i,t,s} \leqslant P_{i,t} \leqslant P_{i,t,down} + (1 - w_{i,t}) P_{i,t,max} \tag{14}$$

### 3.2. Backward dynamic programming (DP)

Dynamic programming is widely used to solve optimisation problems when the problem can be broken down into smaller sub-problems. DP searches the solution space that consists of the unit status for an optimal solution. The search can be carried out in a forward or backward direction. The advantage of DP ist the ability to maintain solution feasibility [49]. It is one of the main solution techniques to optimise the thermal unit commitment schedule [51]. The so-called optimality principle of Bellman [15] states that for an optimal decision sequence, whatever the initial state and the first decision was, the remaining decisions must also form an optimal decision sequence, considered across all possible decision sequences, the beginning of which is the state that results from the first decision.

The implementation is made in Julia, a newly developed programming language which has its strengths in handling numerical problems and data science. The thought-provoking impulse was delivered by [52] which compares different programming languages and shows that Julia challenges well-known established languages, such as R and Python. The open-source project is maintained by a huge community and there is a huge effort in implementing new libraries and features.

Mathematically, the DP approach is explained by Eq. (15) as described by [30], where $T$ is the time horizon; $L$ the number of feasible states; $F_{cost}(T, I)$ the total cost; $P_{cost}(T, I)$ the production cost; $S_{cost}(T-1, L{:}\ T, I)$ the transition cost; and $F_{cost}(T-1, L)$ the fixed cost. Firstly, the state space for the DP algorithm is defined and then derive the corresponding equations basend on the described state space. In general, the backward dynamic programming algorithm inspects every possible state in every interval in the form of an adjacency list for each state, see [53].

$$F_{cost}(T, I) = min[F_{cost}(T, I) + S_{cost}(T-1, L{:}\ T, I) + F_{cost}(T-1, L)] \tag{15}$$

The implemented algorithm differs from the basic form in that it avoids the calculation of the adjacency list for every state. Only reasonable states are calculated, and the calculation overhead is thereby reduced. For this to be possible, a new method called state prediction is developed. This method estimates the next most likely states. In principle, the basic process can be broken down into the following steps, as shown in Fig. 3.

1. Read input data and adjust the data set for faster calculation (linearization and converting special data types for faster operations).
2. Calculate the state cost for $t = T$. The transition cost is zero for the last hour, since it is the initial step. Save the minimum cost value.
3. Calculate the total minimum cost for the previous hour. Now the transition cost is the minimum cost of the previous hour. If there already exists an optimal path to this state, then use the stored information in the memory and skip calculation. Use state prediction to find the next state with the highest probability of minimal costs to reduce calculation steps.
4. Repeat Step 3 until the first hour to obtain the optimal schedule. Allow only paths that are connected to the given state in the initial time step.

The goal of backward dynamic programming is to find one or more paths to the optimal solution. The number of possible paths increases rapidly with increasing time steps. Due to a high number of allowed neighbours from one node, there are many possible paths. On this basis, it can be concluded that the number of possible transitions from one node to the next has to be kept as small as possible to keep the computation time low. Fig. 4 illustrates the structure and operation of the state diagram.

#### 3.2.1. State prediction

In general, state prediction can be explained as follows: When determining the possible previous states at runtime, it is also possible to

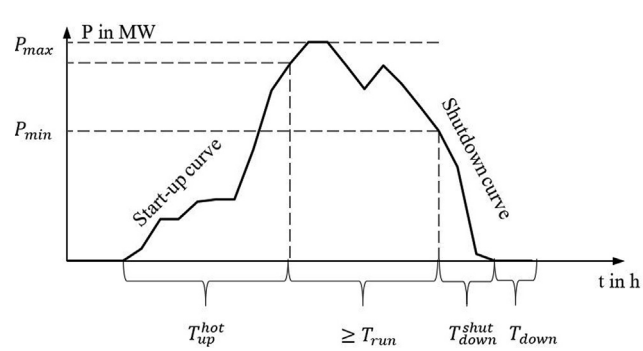


**Fig. 1.** Demonstration of a start-up and shut-down process to reach and leave the minimum power level, adapted from [50].

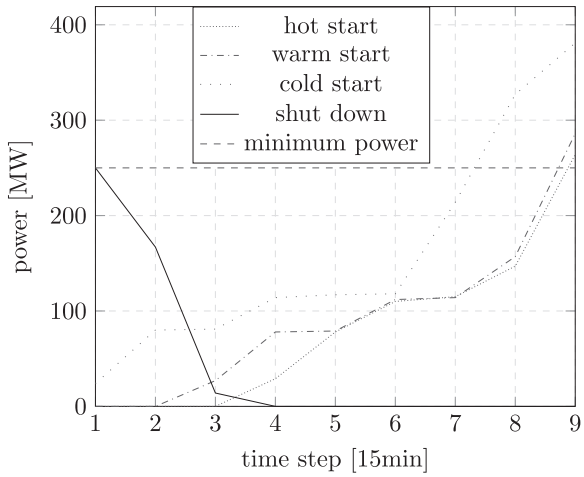


**Fig. 2.** Different start-up and shut-down processes of the power plant unit.

determine directly which transitions to them are the most likely. This method uses the information on market price and ramping procedures that is already known to make a rudimental prediction of the preceding state. These predictions are ordered and weighted according to their probability, thus computing the path that is most likely to be cost-optimal first. The main advantage is that many possible transitions are only calculated when no previous possible transition provides a solution. The introduced state prediction significantly shortens the computation time of optimisation, since many suboptimal paths are not calculated in the first place. The result is verified by comparison with the DP approach without state prediction, leading to the same solution path but with increased computation time. Experience shows that if the unit is either on or off and it is easy to decide whether to keep the unit on or off. As for the algorithm, the state prediction has minor to no impact on computation time in such situations. This can be the case due to extremely high or low market price levels. If there is more uncertainty in the system because of volatile market prices in addition to the opportunity of a ramping process, state prediction significantly enhances performance. The prediction works quite well in most cases and saves overhead computation time.

## 4. Results

The following section presents the results of the previously described solution strategies. The focus lies on the calculated optimal schedules under the perspective of the least needed amount of time. Attention is paid to the relationship between the number of steps, i.e., the planning horizon, and the duration of the computation. The CCGT unit is modelled to reproduce real conditions as closely as possible. The year is modelled in a 15-min time grid that leads to 35,040 steps.

### 4.1. Mixed-integer linear programming

Once the optimisation problem has been solved, Gurobi provides a schedule for the thermal power plant unit. Fig. 5 shows the optimal schedule with the corresponding market price and marginal costs which are achieved by the first derivative of the cost function Eq. (5) with respect to the electrical power output, shown by Eq. (16).

$$mc_t = p_t^{fuel} \cdot q_1 + p_t^{markup} \tag{16}$$

In case of high market prices, attempts are made to stay at the maximum electrical power output, which is equal to the maximum technical power. If the maximum technical power is limited, the power output must also be reduced in the respective duration of time, as shown in Fig. 6. The power output must not exceed the limit of the maximum technical power at any time. The limitation due to the technical minimum output must be observed analogously to the maximum output. These two parameters span the power band in which the electrical power may change as long as the power plant unit is in the online status. In the case of a ramp, this performance band is left.

As soon as the market price falls below the marginal costs, the power output is reduced. The amount of the reduction depends on the marginal costs. At low market prices, the optimisation may decide to shut down the power plant unit. As a result, the power plant unit is brought to a standstill in a short time and there are no further running costs. A new launch in form of a ramp incurs start-up costs that depend on the previous downtime. This aspect is important for the optimisation, since under certain circumstances, the power plant block is kept in operation for a longer period in order to have shorter downtimes. As a result the power plant unit can be started again as soon as possible, thus incurring lower start-up costs. The maximum power change gradient must be observed under all circumstances, except during a start-up or shut-down ramp.

The computation time for the optimisation using the mixed-integer linear method can vary widely. The so-called maximum allowed relative gap has a major influence on the computation time. The gap describes the percentage difference between the upper and lower bounds of optimization based on the branch-and-bound approximation method. Since it is an approximation method, the smaller the relative gap, the closer the result is to the actual optimum. For comparison purposes and in order to keep the computing time low, the relative gap is fixed at 2%. A relative gap of 0,1% is widely used. This results in the computation times shown in Table 1 and the revenues earned for the respective number of steps.

Due to the relatively long computing times and memory shortages, a maximum period of three months is specified for mixed-integer linear programming. As mentioned before, the computation time depends on the market depth. If the market depth is high, the market price is at a point where the threshold is exceeded in order to put the power plant into operation and the power plant is started. A weak market depth means that the state of the power plant is either at a standstill or at maximum output as a result of the current market prices.

The relationship between the number of steps and the computation time is shown graphically in Fig. 7. The trend line illustrates a non-linear relationship, which is a known fact in the case of mixed-integer linear programming. The impact of the market depth of the power plant unit can be clearly seen in the graphic on the basis of those points that deviate significantly from the trend line.

### 4.2. Dynamic programming

The backward dynamic programming provides a cost-optimal schedule for the power plant unit, which can be seen in Fig. 8.

The schedule is calculated using the same parameters as in mixed-integer linear programming. As an additional feature, which was not taken into account in the mixed-integer linear programming, the history of the power plant unit is considered. If the power plant unit is already in a ramp process before the chosen planning horizon, it must be driven until it is finished and the minimum operating time must also be observed before it can be brought to a standstill again. This behaviour can be seen in Fig. 9. Furthermore, the consideration of stronger ramping constraints has a positive impact on all the formulations by reducing their integrality gap and CPU times. Moreover, the consideration of start-up/shut-down trajectories reduces the integrality gap of the problem. It also increases the non-zero elements, which leads to an increase in CPU time [13]. This behavior has also been analyzed in [49] , where the authors conclude that fewer binary variables do not imply less CPU time because the UC formulation may have an increased number of non-zero elements that make the problem more difficult to solve.

The computation time of the model with backward dynamic programming is always linear and depends on the number of steps. The

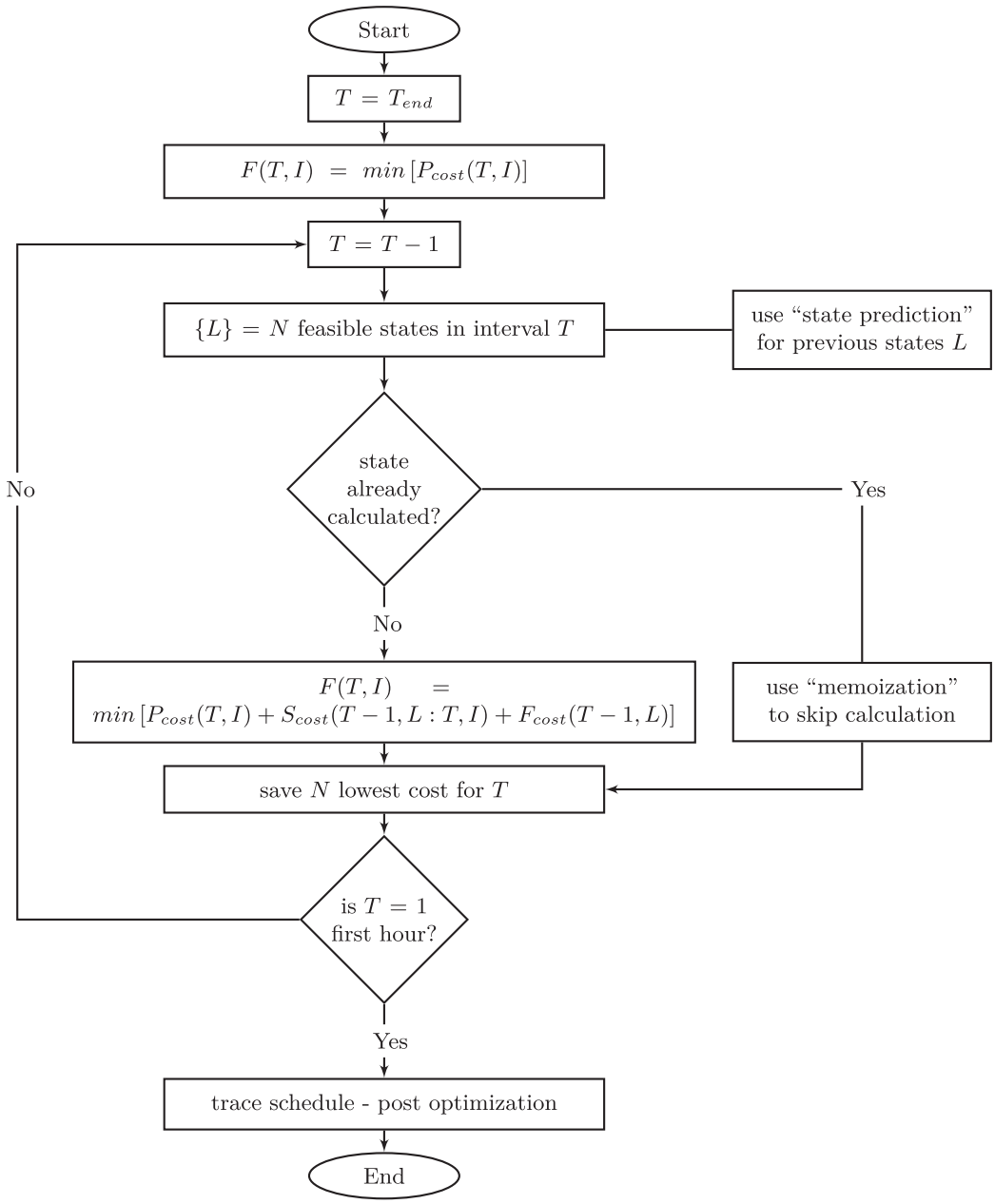


**Fig. 3.** Flowchart for backward dynamic programming. The fundamental principle, based on [30], is extended by a memoization and state prediction algorithm.

data situation has no influence on the computing time of the optimisation problem. Furthermore, there is no duality gap as is the case in mixed-integer linear programming. Nevertheless, it is not an exact optimum, since the calculations are simplified in order to keep the computing time short. In Section 5, considerable simplifications and their influence on the results are described and discussed. Table 2 shows all important results of the backward dynamic programming approach.

In contrast to mixed-integer linear programming, the period for the calculation was increased from three months to one year, since this period can be calculated without running out of memory. Dynamic programming provides a linear relationship between computing time and the number of steps. Fig. 10 illustrates this behaviour.

## 5. Sensitivity analysis

This Section examines the variation of the market price and the influences on the computation time. Moreover, other aspects and simplifications that can influence the computation time are highlighted.

The data situation has a significant influence on the duration of the computation. During this study, it was observed that, depending on the market price, the computation time can vary considerably. In times of extreme market situations, caused for example by very low or very high market prices, the model can be solved in a shorter time than in other market situations. The reason for that is that it is clear to the state prediction algorithm that the optimal schedule dictates either a standstill or maximum technical power. When the market price is in the range of the marginal costs, the optimisation requires more time to decide which power step would be optimal. Fig. 11 show this behaviour.

A major influence on the computation time within the MILP approach is the so-called maximum relative gap. The gap describes the percentage difference between the upper and lower bounds of optimisation on the basis of to the branch-and-bound method. Since it is an approximation method, the result is closer to the actual optimum, the smaller the relative gap.

Hydro or thermal storages often occur in resource planning,

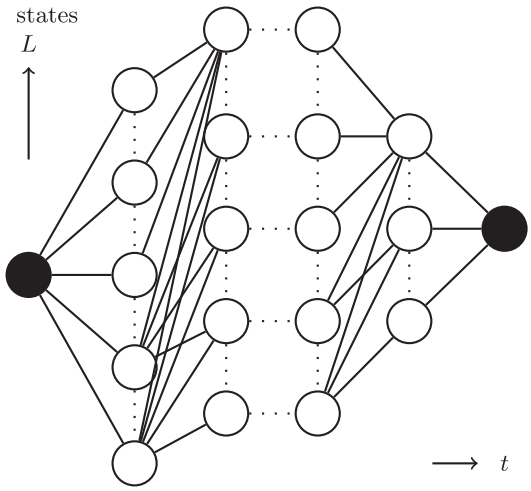


**Fig. 4.** State diagram (directed graph): All states correspond to a power level. Each state transition in the graph represents a decision from one state to another (directed arcs). The source states at $t = T$ and $t = 0$ needs to be selected. All possible transitions between two states are added, as long as they satisfy the constraints (minimum-up/-down time, capacity restrictions and ramping trajectories).

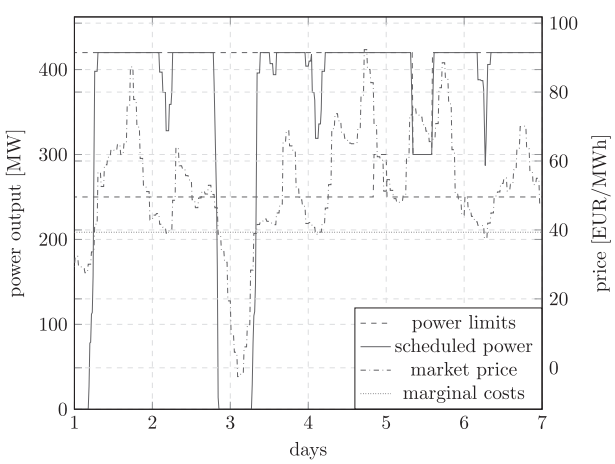


**Fig. 5.** Optimal schedule of the power plant unit for the planning horizon of 01 January 2017 until 08 January 2017, which is determined by MILP.

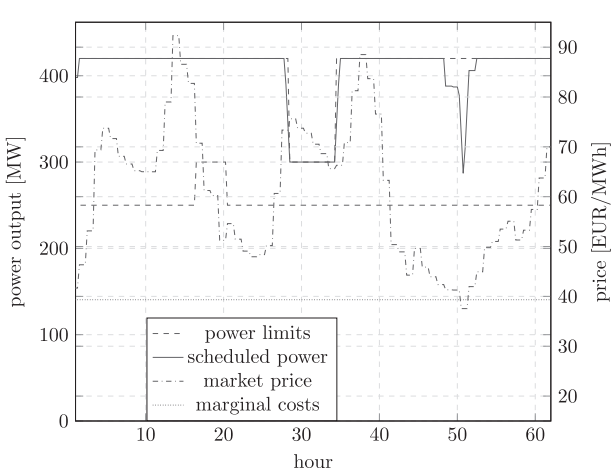


**Fig. 6.** Reduction of the electrical power due to the reduced technical maximum power.

**Table 1**
Computation time and revenue of the MILP approach.

| planning horizon | steps | computation time in sec. | cumulated revenue in EUR |
|---|---|---|---|
| 1 day | 96 | 4,666 | 0 |
| 1 week | 672 | 43,932 | 449 885 |
| 2 weeks | 1 344 | 254,4 | 983 782 |
| 3 weeks | 2 016 | 157 | 3 413 456 |
| 1 month | 2 688 | 350 | 5 228 315 |
| 5 weeks | 3 360 | 557 | 5 959 106 |
| 6 weeks | 4 032 | 988 | 6 741 175 |
| 7 weeks | 4 704 | 1 027 | 7 245 565 |
| 2 months | 5 376 | 1 122 | 7 245 565 |
| 3 months | 8 064 | 2 581 | 7 653 258 |

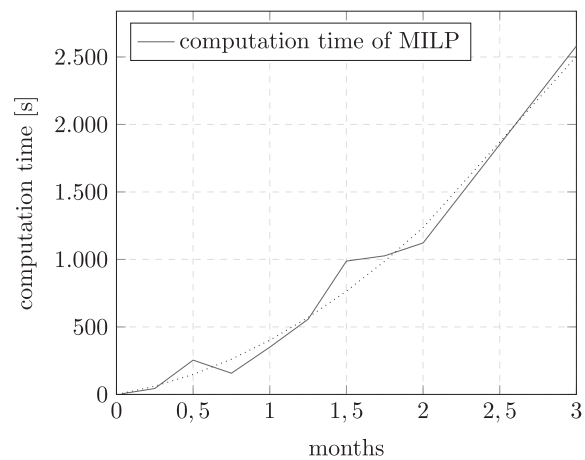


**Fig. 7.** Computation time depending on the number of steps. The dotted trend line illustrates the non-linear relationship.

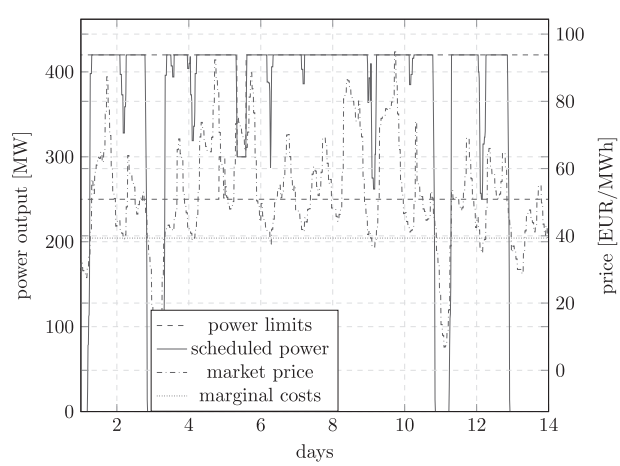


**Fig. 8.** Schedule of the electrical power dispatch from 01 January 2017 to 15 January 2017 which was determined by backward dynamic programming with state prediction.

however, they were outside the scope of this examination. It is undisputed that the implementation of storages increases the level of complexity even further and will lead to the so-called curse of dimensionality in the DP approach due to several choice problems in the unit commitment [15]. At a certain point, the DP strategy needs more amount of computation time and may take longer than a MILP approach.

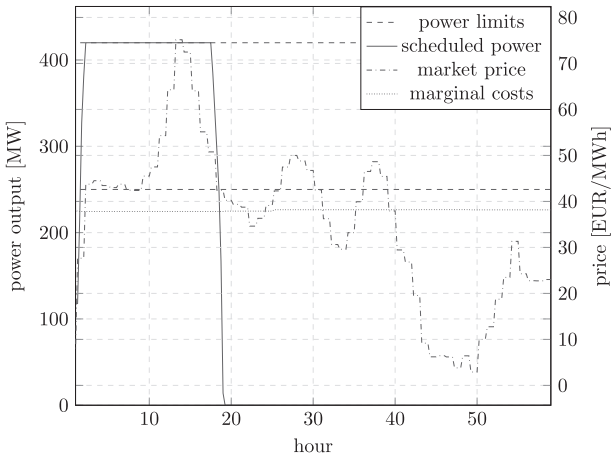


**Fig. 9.** Completion of a ramp and compliance with the minimum operating time before the power plant unit can be shut down.

**Table 2**
Result of the dynamic programming approach with state prediction regarding computation time, required memory and cumulated revenue.

| planning horizon | steps | computation time in sec. | required memory in MB | cumulated revenue in EUR |
|---|---|---|---|---|
| 1 day | 96 | 0,18 | 13 | 0 |
| 1 week | 672 | 0,25 | 27 | 455 783 |
| 2 weeks | 1 344 | 0,29 | 45 | 941 895 |
| 3 weeks | 2 016 | 0,30 | 49 | 3 455 730 |
| 1 month | 2 688 | 0,35 | 61 | 5 287 260 |
| 5 weeks | 3 360 | 0,40 | 83 | 6 027 900 |
| 6 weeks | 4 032 | 0,44 | 101 | 6 898 790 |
| 7 weeks | 4 704 | 0,48 | 118 | 7 376 080 |
| 2 months | 5 376 | 0,59 | 160 | 7 360 440 |
| 3 months | 8 064 | 0,86 | 284 | 7 773 110 |
| 4 months | 10 752 | 1,14 | 422 | 8 085 080 |
| 6 months | 16 128 | 1,67 | 664 | 8 716 790 |
| 8 months | 21 504 | 2,15 | 850 | 10 415 500 |
| 9 months | 24 192 | 2,33 | 953 | 10 973 900 |
| 1 year | 35 040 | 3,72 | 1 515 | 16 604 600 |

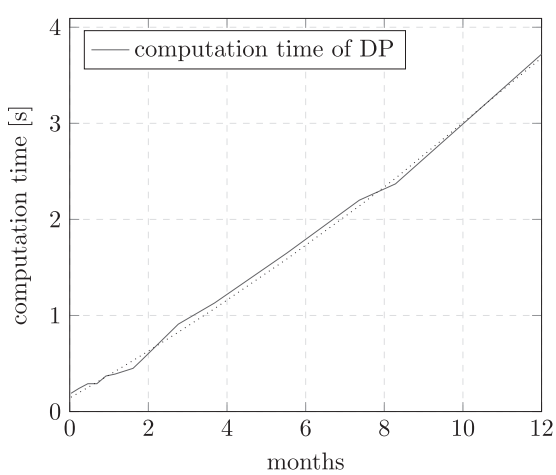


**Fig. 10.** Computation time depending on the number of steps in backward dynamic programming. The dotted trend line illustrates the linear relationship.

## 6. Discussion and synthesis of results

In this section, the previously presented results are compared with each other, and a statement is made regarding the advantages and disadvantages of the respective solution variant. In Section 6.1, the

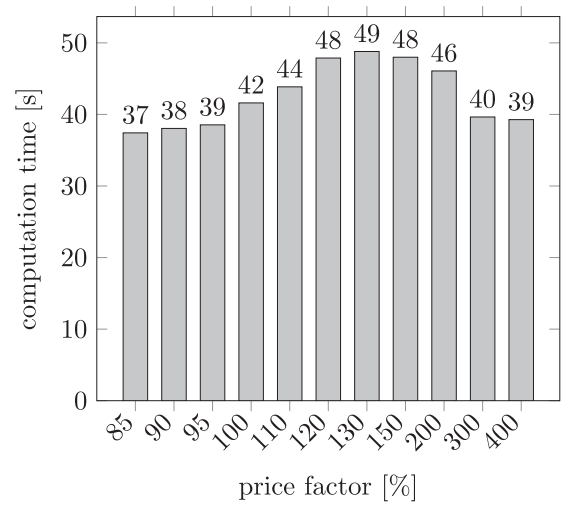


**Fig. 11.** Sensitivity analysis of the computation time in terms of the electricity price factor for a period of seven days for the MILP approach.

**Table 3**
Comparison of the MILP and DP + SP approaches. It can be clearly seen that the backward dynamic programming approach with state prediction outperforms the MILP approach in terms of computation time and memory requirement.

| planning horizon | steps | computation time in sec. | | required memory in MB | |
|---|---|---|---|---|---|
| | | MILP | DP + SP | MILP | DP + SP |
| 1 day | 96 | 4,666 | 0,18 | 54 | 13 |
| 1 week | 672 | 43,832 | 0,25 | 201 | 27 |
| 2 weeks | 1 344 | 254,4 | 0,29 | 440 | 45 |
| 3 weeks | 2 016 | 257 | 0,30 | 753 | 49 |
| 1 month | 2 688 | 350 | 0,35 | 1 139 | 61 |
| 5 weeks | 3 360 | 557 | 0,40 | 1 599 | 83 |
| 6 weeks | 4 032 | 988 | 0,44 | 2 132 | 101 |
| 7 weeks | 4 704 | 1 027 | 0,48 | 2 739 | 118 |
| 2 months | 5 376 | 1 122 | 0,59 | 3 419 | 160 |
| 3 months | 8 064 | 2 581 | 0,86 | 6 873 | 284 |
| 4 months | 10 752 | – | 1,14 | – | 422 |
| 6 months | 16 128 | – | 1,67 | – | 664 |
| 8 months | 21 504 | – | 2,15 | – | 850 |
| 9 months | 24 192 | – | 2,33 | – | 953 |
| 1 year | 35 040 | – | 3,72 | – | 1 515 |

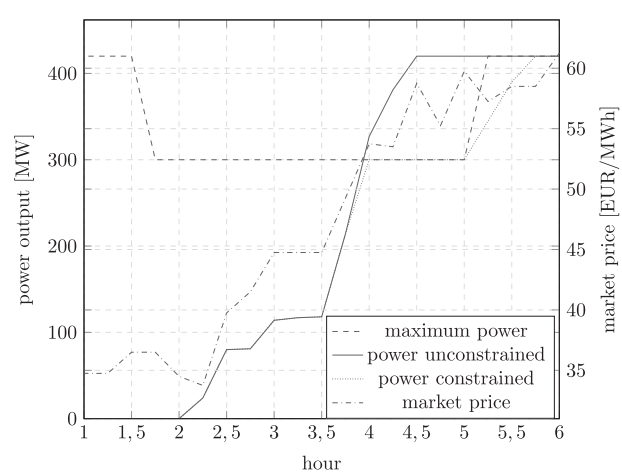


**Fig. 12.** Modification of the cold start ramp due to the limitation of the electrical call power by the technical maximum power.

**Table 4**
Validation of results by comparing the implemented approach with an already existing working algorithm of a generating company.

| planning horizon | steps | Total revenue in EUR | | |
|---|---|---|---|---|
| | | MILP | DP + SP | cDP |
| 1 day | 96 | 0 | 0 | 0 |
| 1 week | 672 | 449 855 | 455 783 | 462 502 |
| 2 weeks | 1 344 | 983 782 | 941 895 | 999 112 |
| 3 weeks | 2 016 | 3 413 456 | 3 455 730 | 3 465 560 |
| 1 month | 2 688 | 5 228 315 | 5 287 260 | 5 302 299 |
| 5 weeks | 3 360 | 5 959 106 | 6 027 900 | 6 040 376 |
| 6 weeks | 4 032 | 6 741 175 | 6 898 790 | 6 924 084 |
| 7 weeks | 4 704 | 7 245 565 | 7 376 080 | 7 404 035 |
| 2 months | 5 376 | 7 245 565 | 7 376 080 | 7 404 035 |
| 3 months | 8 064 | 7 653 285 | 7 773 110 | 7 883 787 |
| 4 months | 10 752 | – | 8 085 080 | 8 161 197 |
| 6 months | 16 128 | – | 8 716 790 | 8 792 760 |
| 8 months | 21 504 | – | 10 415 500 | 10 473 222 |
| 9 months | 24 192 | – | 10 973 900 | 11 097 987 |
| 1 year | 35 040 | – | 16 604 600 | 16 763 020 |

**Table 5**
Comparison of the calculated profit and the resulting difference to the estimated optimal schedule.

| case | revenue in EUR | deviation of optimum in % |
|---|---|---|
| cDP | 16 863 020 | 0,40 |
| DP + SP | 16 604 615 | 0,49 |
| difference | 258 405 | 0,09 |

MILP and DP + SP (dynamic programming + state prediction) approaches for solving the optimisation problem are compared. Section 6.2 focuses on the verficiation of the results by comparing the DP + SP approach with state prediction with a cDP (classic DP) algorithm formulated and used by an utility.

### 6.1. Comparison of MILP and DP + SP approach

Table 3 summarises the results and shows the direct comparison of the two approaches in terms of computation time and memory requirement. Within the MILP approach, only three months could be solved otherwise, the installed memory would have been exceeded.

There is no doubt that backward dynamic programming has an advantage over mixed-integer linear programming in computation time. The higher the number of steps, the larger the time difference becomes. In addition, the computation time for mixed-integer linear programming scales disproportionately with an increasing number of steps. As a result, either the size of the problem must be kept small or the time granularity as large as possible.

By expanding the model with more constraints, such as specific ramp curves, the complexity of the model increases, which strongly impacts computation time. Compared to backward dynamic programming, where extensions often only require less additional effort in the computation, certain requirements cannot be formulated using mixed-integer linear programming or can only be implemented only with great difficulty. The combination of a cold start and a reduced technical maximum output is an example of this. If the maximum technical power is below the final power value of a cold start, a non-permitted situation occurs and therefore it cannot be driven. Due to the just-started downtime, only a cold start ramp is an option. However, the end of the ramp is limited by the maximum technical performance. The dispatched power must be below this limit under all circumstances and therefore cannot achieve the cold start process while remaining at the maximum technical power. After the limitation is lifted, the power plant unit can resume normal operation. This fact can be implemented in the backward dynamic programming, in which the cold start is limited by the technical maximum performance and these states are allowed. Fig. 12 illustrates this behaviour.

It is worth mentioning that utilities will handle such market situations manually, since the optimization delivers a less than optimal schedule. In the case of dynamic programming, no manual intervention would be required. There is no doubt that a manual check would still be necessary, since it cannot be assumed that the algorithm works error-free in all market situations at all times and always guarantees an optimal schedule.

### 6.2. Verifying the results

The total revenue shown in Table 4 and displayed in Fig. ?? can be considered a validation of the results. The MILP and DP + SP approaches are compared with the cDP approach to ensure the strategies fulfil their intended purpose. The cDP results are provided by an existing dynamic programming algorithm from a local generating company.

Due to the differences in the algorithms, both approaches provide slightly different solutions. The schedules are relatively identical and differ only marginally. The DP + SP allows the actual drive of each power value. The cDP solution, on the other hand, works with a fixed grid of power values that can be jumped to. Differences in earned revenue can also result from the fact that at certain periods the power plant unit is kept in operation even at low market prices and is only shut down later. The reason for this lies in the lower start-up costs, since under certain circumstances a warm start can be carried out instead of a cold start.

Since both solutions are purely approximate, it is only possible to estimate the deviation from the acutal optimal schedule. On the basis of the marginal costs and neglect of the change in maximum power gradient, an optimal schedule can be calculated. In reality, this is undoubtedly not driveable due to technical restrictions.. Attempts are made to get as close as possible to the optimal schedule, taking the technical restrictions into account by using the best possible approximations. This calculation can be used to make a statement about the quality of the optimisation by calculating the deviation at each individual point in time and averaging over all time steps. Table 5 shows

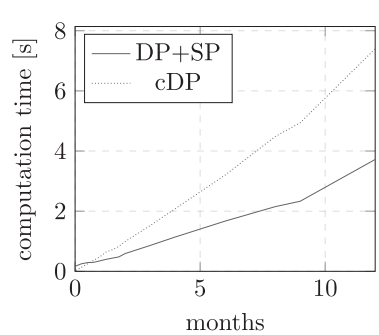


**Fig. 13.** Market price of the Energy Exchange Austria from 01 January 2017 to 31 January 2017 in a 15 min time grid.

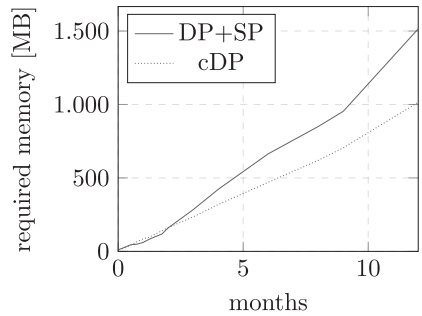


**Fig. 14.** Comparison of the computation time of the DP + SP and cDP approaches.

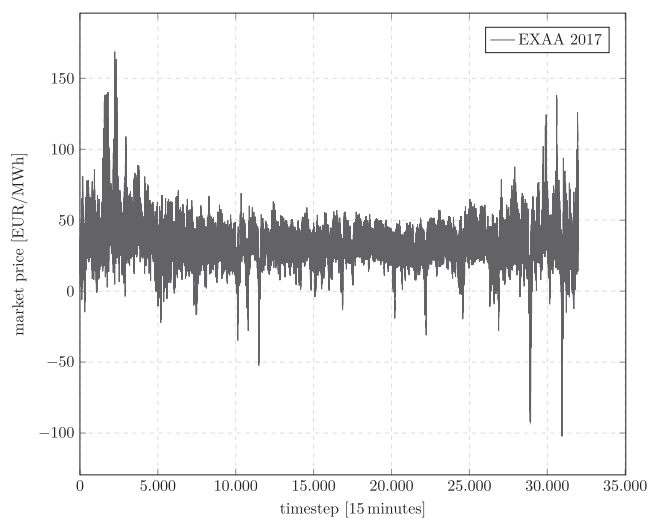


**Fig. 15.** Comparison of the memory requirement of the DP + SP and cDP approaches.

the comparison of the generated profit of both solution variants for the period of one year.

The DP + SP algorithm requires half as much time as the cDP approach. In this sense, the solution with state prediction has a performance advantage over a classic algorithm. The implementation of the state prediction algorithm proves especially worthwile when the number of steps increases, since the computation time is significantly reduced. Figs. 14 and 15 illustrate the comparison of the computation time as well as memory requirement of the two approaches. The memory requirements of the approaches are low and thus be neglected.

## 7. Conclusions

The optimal unit commitment of thermal systems results in enormous savings for electrical utilities and is therefore essential for modern economic dispatch. In this paper the focus is put on improving computational performance to solve the deterministic profit based single-UC problem. To achieve this goal, an efficient dynamic programming algorithm and linear program formulations is proposed. Time-dependent start-up costs as well as start-up and shut-down trajectories in the formulations which improves tightness but increases non-zero elements, which leads to an increase in computation time are considered.

A more efficient dynamic programming algorithm by formulating a new extension called state prediction is proposed. The results show, that taking state predicition into account reduces the computational burden significantly, while simultaneoulsy improving solution quality. Furthermore, the given linearity of the dependence of computation time and the number of states is a significant advantage of the proposed strategy. The formulation is gets far more computationally effective as planning horizon grows. Besides solving single-unit self-scheduling/bidding problem, the extended formulation could potentially help speeding up approaches to solve the multi-UC problems efficiently. While the formulation is only tested on a particular test system, the newly developed approach opens up perspectives for the treatment of a more complex UC model for increased industrial relevance, where similar results can be expected. This may constitute the object of future studies.

Following up on this topic, future research may seek to examine how well the state prediction would operate if additional constraints, such as operating reserve constraints, transmission constraints, emission constraints, reactive power constraints and further constraints, are added. Future research should further develop and confirm these initial findings by extending the proposed deterministic formulation to stochastic UC formulation due to increasing RES in power supply and the resulting uncertainties. Since this research focuses on accurate formulations of ramping behaviour for CCGT units, a more accurate approach could be examined by adding an edge-based modeling framework for CCGT units which is desirable for future work.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix A

### A.1. EXAA market prices

Fig. 13 illustrates the EXAA market prices in 2017. During several time intervals, a negative market price was registered. The range lies between −102,34 EUR/MWh and 168,85EUR/MWh with an average market price of 34,53 EUR/MWh.

Figs. 14 and 15.

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