Efficient Demodulation for Measuring the Amplitude of Mechanical Oscillations

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***Abstract*—The accurate detection and demodulation of mechanical oscillations is crucial for the performance of miniaturized resonant sensors and scanners. Demodulation by external instruments such as lock-in amplifiers or spectrum analyzers is not always desired due to their large size, complexity and cost. The demodulation technique proposed in this paper enables a simplified measurement of the oscillation amplitude of a mechanical oscillator with integrated deflection sensors, such as piezoresistive or capacitive elements. Configuring the sensors in an AC bridge circuit operated at the oscillator resonance frequency leads to a direct demodulation at the output of the bridge circuit. The presented method is experimentally verified on an AFM cantilever with integrated piezoresistive elements.**

***Index Terms*—Demodulation, Resonant sensor, Cantilever, AFM**

I. INTRODUCTION

Micrometer scale mechanical oscillators are employed in a variety of transducer applications. For instance, micromachined cantilevers are used for different measurement modes in Atomic Force Microscopy (AFM) [1], as well as numerous biological [2], [3], chemical [4] and environmental [5] sensing systems. Resonant MEMS mirrors are used for beam steering in optical scanning systems, such as light detection and ranging (LIDAR) [6], and various other medical applications [7], [8]. In the mentioned sensing applications, the cantilever is typically excited at its resonance frequency by an acoustic, thermal or electrical stimulus [3]. The interaction with the environment leads to a shift of the resonance frequency, which translates to a modulation, i.e. a low frequency variation of amplitude or phase, of the oscillation [9], [10]. The detection and demodulation of the cantilever oscillation is therefore a crucial part of these sensor systems. Similarly, the ability to measure and control the oscillation of resonant mirrors can determine the accuracy of optical scanning systems.

Depending on the application, different detection schemes are applied, such as optical, capacitive and piezoresistive detection [4]. Optical detection is widely used for cantilever deflection measurements due to its high sensitivity which enables a reliable detection of deflections in the sub-nanometer range [11]. However, optical detection has several important limitations. Due to optical diffraction and the resulting finite optical beam diameter, the minimum size of the cantilever is limited to a few micrometers. It also requires a cumbersome laser alignment process and the entire system is rather bulky and costly. In capacitive and piezoresistive detection schemes the mechanical motion is translated to a variation of the impedance of integrated elements on the oscillator [12], [13]. The impedance variation is typically converted to electrical signals by a read-out electronic integrated on the chip, which enables mass-produced and inexpensive devices. Additionally, the highly compact detection enables an easy extension to cantilever arrays [14] or parallel probing systems [15].

The output of the mentioned detection methods is an electrical signal proportional to the deflection of the mechanical oscillator. The oscillation therefore has to be demodulated in order to determine the deflection amplitude or phase. In most transducer applications external instruments, such as lock-in amplifiers [6], [16], [17], spectrum analyzers [18] or impedance analyzers [19] are used for demodulation. They provide a narrow-band measurement of amplitude and phase, which is insensitive to other frequency components. However, due to the high frequencies of micrometer scale oscillators of up to tens of MHz, these techniques have a high implementation complexity and are therefore bulky and expensive. This limits portability, compactness and price of the overall transducer systems.

The contribution of this paper is the proposal and implementation of an efficient demodulation method to measure the amplitude of mechanical oscillations. The presented method can eliminate the need for bulky and costly external demodulation techniques in many applications. After a system description and the review of the working principle of a conventional lock-in amplifier in Section II, the proposed method is presented in Section III. After discussing the experimental implementation in Section IV, the experimental results are shown in Section V, followed by a conclusion in Section VI.

II. SYSTEM DESCRIPTION

A self-sensing AFM cantilever of the type PRSA-L300-F50-Si-PCB (SCLSensortech, Vienna, Austria) with a resonance frequency of $\omega_0 = 2\pi \cdot 43.6\,\text{kHz}$ is used. As illustrated in Figure 1 the cantilever has two integrated piezoresistive elements with a resistance of $R = 1.07\,\text{k}\Omega$ at its base. The piezoresistive elements are configured in a half bridge circuit to detect the small change of the resistance due to the cantilever deflection. The two remaining resistors of the bridge circuit are integrated on the AFM chip. An optical image of the

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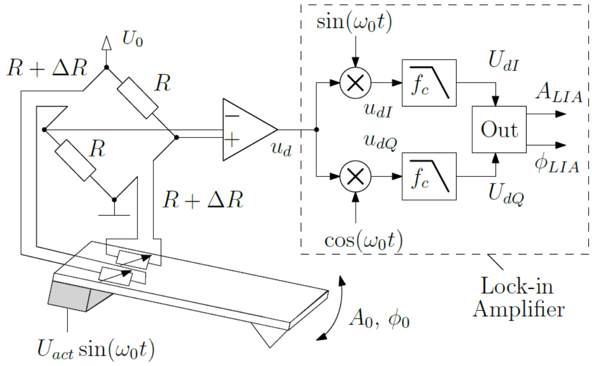

Fig. 1: Illustration of an AFM cantilever with two piezoresistive elements at its base, which are configured in a half bridge circuit. The cantilever is mechanically excited at its resonance frequency ω_0 by a piezoelectric actuator, and the oscillation is demodulated by a lock-in amplifier.

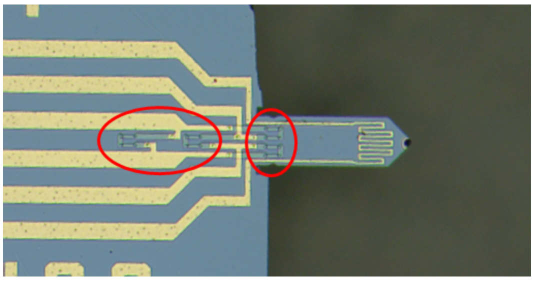

Fig. 2: Optical image of the used self-sensing AFM cantilever with two piezoresistive elements at its base and two additional resistors integrated on the AFM chip (SCLSensortech, Vienna, Austria).

self-sensing cantilever is shown in Figure 2, where the piezoresistive elements on the cantilever (right) and the chip (left) are highlighted. A piezoelectric actuator (PhysikInstrumente, Karlsruhe, Germany) mechanically excites the cantilever at the frequency ω_0. The resulting variation of the resistance can be expressed as

$$\Delta R = A_0 K \sin(\omega_0 t + \phi_0)\,, \tag{1}$$

with oscillation amplitude A_0 and phase ϕ_0, as well as the piezoresistive sensitivity K [21]. Assuming $\Delta R \ll R$, the resulting bridge output voltage u_d equals

$$u_d(t) \approx \frac{U_0 K}{2R} A_0 \sin(\omega_0 t + \phi_0)\,, \tag{2}$$

where U_0 and R are the supply voltage of the circuit and the nominal resistance of the piezoresistive elements, respectively. In order to determine A_0 or ϕ_0, the voltage u_d has to be demodulated.

A. *Conventional Demodulation by Lock-In Amplifier*

In this section, the working principle of a lock-in amplifier is reviewed as conventional method. The proposed demodulation method is presented in the following section.

Figure 1 shows the working principle of the demodulation by a lock-in amplifier. The bridge output voltage u_d (2) is multiplied by in-phase and quadrature sinusoidal signals. The resulting voltages

$$u_{dI}(t) = \frac{U_0 K}{4R} A_0 \left[\cos(\phi_0) - \cos(2\omega_0 t + \phi_0)\right]\,, \tag{3}$$

$$u_{dQ}(t) = \frac{U_0 K}{4R} A_0 \left[\sin(\phi_0) + \sin(2\omega_0 t + \phi_0)\right] \tag{4}$$

are applied to low-pass filters to suppress the spectral components at $2\omega_0$ and obtain

$$U_{dI} = \frac{U_0 K}{4R} A_0 \cos(\phi_0)\,, \tag{5}$$

$$U_{dQ} = \frac{U_0 K}{4R} A_0 \sin(\phi_0)\,. \tag{6}$$

The amplitude A_{LIA} of the voltage u_d can be calculated by

$$A_{LIA} = 2\sqrt{U_{dI}^2 + U_{dQ}^2} = \frac{U_0 K}{2R} A_0\,, \tag{7}$$

which is equal to the oscillation amplitude up to a constant factor. The oscillation phase can directly be calculated by

$$\phi_{LIA} = \arctan\left(\frac{U_{dQ}}{U_{dI}}\right) = \phi_0\,. \tag{8}$$

The lock-in amplifier enables a demodulation of the oscillation which is insensitive to noise components outside of the frequency range defined by the low-pass filters. The crossover frequency of the low-pass filters is therefore determined in a trade-off between low noise and high demodulation bandwidth.

In today's transducers, lock-in amplifiers are typically implemented on digital signals processors (DSPs). Due to the complexity of the digital implementation, the minimum number of samples per oscillation period, as well as the required suppression of the 2ω-component, the maximum oscillation frequency which can be demodulated is 10 to 100 times lower than the sample rate of the DSP [22]. For resonance frequencies of tens of MHz, analog to digital converters (ADCs) with sample rates beyond 100 MHz [17] can be required. The size, cost and complexity of ADCs and lock-in amplifier can therefore by far exceed those of the micromachined oscillator itself, which presents a severe limitation of the overall transducer system.

III. PROPOSED DEMODULATION METHOD

Figure 3 shows the proposed method for a simplified demodulation. The bridge circuit is supplied by an AC-voltage with frequency ω_0 and an adjustable phase ϕ_c. Therefore, the resulting bridge output voltage equals

$$u_{d,AC}(t) = \frac{U_0 K}{4R} A_0 \left[\cos(\phi_0 - \phi_c) - \cos(2\omega_0 t + \phi_0 + \phi_c)\right]\,. \tag{9}$$

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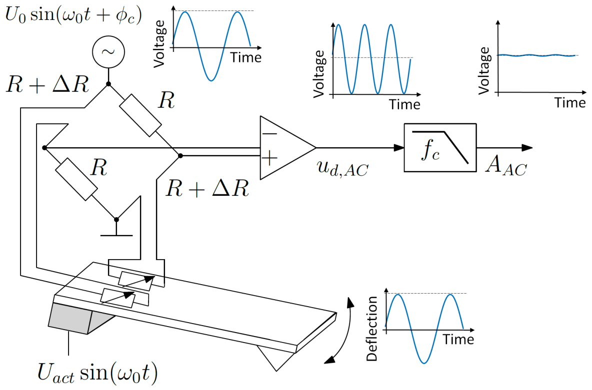

Fig. 3: Proposed demodulation. The bridge circuit is supplied by an electrical signal with the same frequency as the mechanical oscillation, which leads to a direct demodulation at the bridge output voltage.

As illustrated by the plotted signals in Figure 3, supplying the bridge circuit by an electrical signal of the same frequency ω_0 as the mechanical oscillation leads to a multiplication of the two signals. The bridge output voltage therefore contains a DC component proportional to the amplitude of the mechanical oscillation, as well as a component at frequency $2\omega_0$. Comparison of (9) with (3) and (4) shows that this demodulation delivers the same result as the multiplication with an external sinusoidal voltage. For a constant phase $\phi_0 = \phi_c$, the oscillation amplitude A_0 can therefore directly be obtained after removing the $2\omega_0$ component by a low-pass filter:

$$A_{AC} = \frac{U_0 K}{4R} A_0 \cos(\phi_0 - \phi_c)\,. \tag{10}$$

Alternatively, for a constant amplitude A_0 and a given phase ϕ_c, the filtered output (10) delivers the phase ϕ_0. The implementation requires no cumbersome external multiplications of electrical signals. The low-pass filter can be easily implemented by passive elements and integrated onto the chip.

The proposed method is not limited to the detection of mechanical oscillations by piezoresistive sensors. For instance, the impedance variation of capacitive sensors could be demodulated by configuring the capacitances in an AC bridge circuit in a similar way.

IV. EXPERIMENTAL IMPLEMENTATION

Figure 4 shows an image of the experimental setup. The self-sensing cantilever and its connector are mounted on a piezoelectric actuator which is glued to an aluminum fixture. The bridge output voltage is amplified by a pre-amplifier with an amplification of 100. For the implementation of the conventional method, the bridge circuit is supplied by a DC voltage of $U_0 = 2\,\text{V}$ and the amplifier output u_d is applied to a lock-in amplifier (Ametek 7270, Pennsylvania, US) with a selected low-pass filter cross-over frequency of 5 kHz. For the proposed demodulation an AC voltage with an amplitude of $U_0 = 2\,\text{V}$ and frequency ω_0 is applied to the bridge circuit. The amplifier output $u_{d,AC}$ is filtered by an analog low-pass filter with a cross-over frequency of 5 kHz. To verify and compare the two methods, the signals u_d and A_{LIA} of the conventional method, as well as $u_{d,AC}$ and A_{AC} of the proposed method, are recorded by an oscilloscope (Agilent DSO-X 2004A, Santa Clara, US). All applied signals are generated by function generators (Agilent 33500B, Santa Clara, US).

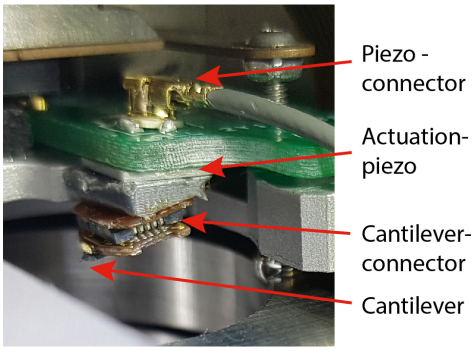

Fig. 4: Experimental setup for the demonstration of the proposed demodulation method.

A. Compensation of bridge circuit imbalance

The integrated bridge circuit is not perfectly balanced. Without deflection of the cantilever, the manufacturer specifies an offset of 0.4 V at the output of the pre-amplifier for a DC supply voltage of 2.05 V. For the proposed method, this imbalance results in an AC voltage proportional to the supply voltage at the amplifier output, which is superimposed on the voltage $u_{d,AC}$ and would lead to a measurement error. However, the imbalance can be compensated by subtracting an AC voltage with a constant amplitude from the amplifier output.

V. EXPERIMENTAL RESULTS

In order to verify the analysis presented in Section III the conventional demodulation by a lock-in amplifier and the proposed demodulation method are compared. The piezoelectric actuator exciting the cantilever is operated at frequency ω_0 with a constant excitation amplitude of $U_{act} = 10\,\text{V}$.

In Figure 5, the frequency components of the measured voltages u_d and A_{LIA} at DC, ω_0 and $2\omega_0$ are shown. As expected from (2), u_d only shows a component at frequency ω_0 (see Figure 5a). The oscillation is converted to a DC voltage A_{LIA} at the output of the lock-in amplifier, which is proportional to the cantilever oscillation amplitude A_0 (see Figure 5b).

Figure 6 shows the frequency components of the voltages $u_{d,AC}$ and A_{AC}. The voltage $u_{d,AC}$ in Figure 6a contains two main components at DC and $2\omega_0$, which is in accordance with (9). The non-zero component at ω_0 can be explained by an imperfect compensation of the bridge circuit imbalance.

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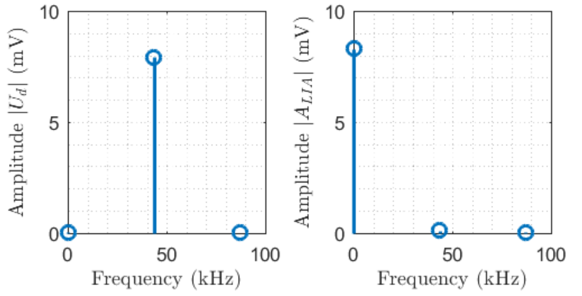
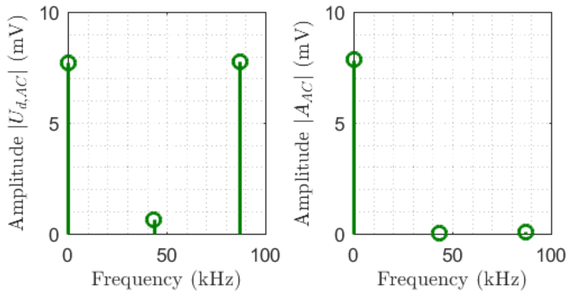


(a) (b)

Fig. 5: Demodulation with lock-In amplifier: spectral components of (a) bridge output voltage $u_d(t)$ and (b) demodulated amplitude A_{LIA} at DC, ω_0 and $2\omega_0$.



(a) (b)

Fig. 6: Proposed demodulation: spectral components of (a) bridge output voltage $u_{d,AC}(t)$ and (b) demodulated amplitude A_{AC} at DC, ω_0 and $2\omega_0$. The DC component of the bridge output voltage is proportional to the cantilever oscillation amplitude, which can therefore directly be obtained by low-pass filtering.

The oscillation amplitude can directly be obtained by low-pass filtering of $u_{d,AC}$. The resulting DC voltage A_{AC} in Figure 6, is equal to the voltage A_{LIA} measured by the conventional method. Note, that the voltage A_{AC} is multiplied by a factor of 2 in software in order to compensate for the different factors of the two demodulation methods (compare (7) and (10)).

For the measurement in Figure 6a, the phase ϕ_c of the bridge supply voltage is manually adjusted, such that the components at DC and $2\omega_0$ are equal, which corresponds to $\phi_c = \phi_0$. Figure 7 shows the measured amplitude A_{AC} depending on the phase ϕ_c of the bridge supply voltage. The resulting sinusoidal relation is in accordance with (10).

In Figure 8, the excitation amplitude U_{act} of the piezoelectric actuator, and therefore the cantilever oscillation amplitude A_0, is varied. The measured amplitudes obtained by conventional and proposed method both show a linear dependence on the actuation voltage, which verifies the analysis in Section III.

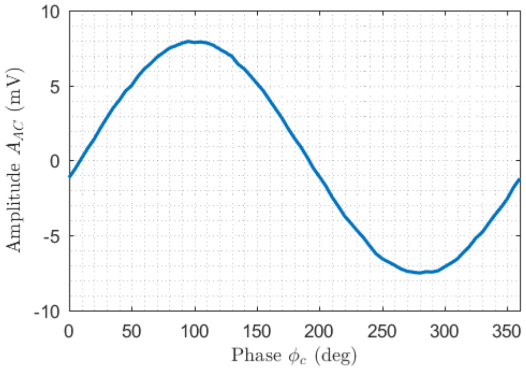


Fig. 7: Demodulated amplitude A_{AC} depending on the phase ϕ_c of the bridge supply voltage.

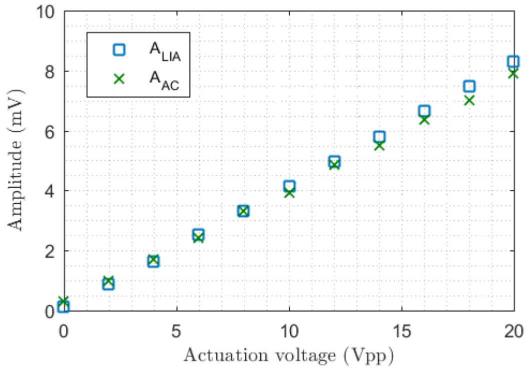


Fig. 8: Comparison of the demodulated amplitudes A_{LIA} and A_{AC}, depending on the piezo voltage U_{act}.

In summary it has been shown that the measurement of the amplitude of mechanical oscillations can be simplified by the proposed demodulation method.

VI. CONCLUSIONS

The presented demodulation method enables a simplified measurement of the amplitude of mechanical oscillations. It has been analytically derived that the configuration of piezoresistive elements in an AC bridge circuit which is operated at the mechanical oscillation frequency leads to a direct demodulation at the bridge output voltage. The oscillation of an AFM cantilever with integrated piezoresistive elements was demodulated to verify the analytic analysis. The proposed method only requires low-pass filters which can easily be integrated on micro-machined oscillators and enables the development of cost-efficient and highly integrated transducers. Ongoing work is focused on the efficient measurement and control of oscillation amplitude and phase in MEMS oscillators and AFM cantilevers.

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