Occupational safety in a frictional labor market

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Abstract

This paper studies the provision of occupational safety when the labor market is subject to search frictions, and the safety level is determined endogenously. While safety measures are costly for firms, they lower workers' mortality. We show that the presence of search frictions leads to excess mortality compared to a frictionless labor market. In a decentralized setting where wages and safety levels are bargained at the firm level, matching externalities and a labor supply externality may lead to further increases in mortality. We obtain conditions under which these externalities are internalized by firms and workers, and discuss the role of policy for promoting occupational safety.

Keywords: occupational safety, mortality, search frictions, wage bargaining

JEL classification: J17, J28, J32, J38, J64

1 Introduction

In light of increasing demographic pressure on public welfare and healthcare systems, policymakers emphasize occupational safety as a key factor for longer working lives and healthy aging. Initiatives like those of the European Commission (2021) and the National Institute for Occupational Safety and Health (2019) are warranted since the level of occupational safety arising from the interplay of firm and worker incentives is likely to be inefficient (Henderson, 1983). This is due to the presence of asymmetric information and psychological biases as well as externalities on co-workers and society that individual firms and workers do not take into account (Pouliakas and Theodossiou, 2013).

Another source of inefficiencies that so far has received little attention in the context of occupational safety provision are labor market imperfections that arise from frictions in the matching of unemployed to job openings. Stronger frictions increase the time that unemployed need to find and take up a job. This reduces the worker's outside option when bargaining with a firm, which potentially results in the acceptance of lower safety standards if the level of occupational safety is determined at the firm-level. Suggestive evidence for this channel

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is provided by Robinson (1988, 1990) who finds that workers in more hazardous workplaces are more inclined to vote for unions, thus trying to increase their impact on workplace safety policies. In a frictionless labor market, these workers could freely move to other employers such that competition between firms leads to efficient safety levels. Since search frictions make quits costly, workers seek other ways to improve their working conditions.[1]

Unions indeed seem to be an important mediator in the provision of workplace safety. Donado and Wälde (2012) argue that in a historical perspective, worker movements have played a crucial role in making workplaces safer by providing more accurate information about occupational risk and raising worker voice.[2] Additionally, unions can help to overcome a labor supply externality that potentially arises when workplace safety is determined at the firm-level. Private negotiations may fail to take into account that a worker dying due to occupational risks is not only lost for the current employer but for *all* firms in the economy, as aggregate labor supply shrinks. In this case, the privately agreed level of occupational measures is below the socially optimal one.

This paper studies the provision of occupational safety in the presence of search frictions as featured in the workhorse model of modern labor economics, the Diamond-Mortensen-Pissarides (DMP) model. Since occupational safety ultimately affects workers' mortality, we extend the basic DMP model (Pissarides, 2000, Ch. 1) for mortality shocks. The mortality rate of employed individuals is endogenously determined and our main variable of interest. We solve three versions of our model to identify (i) the effect of search frictions and (ii) the effect of externalities.

We find that the presence of search frictions increases the socially optimal mortality rate. The socially optimal level trades off the current costs of safety measures with their long-term benefits. The latter accrue from worker's higher life expectancy, which translates into higher lifetime income and utility. Phases of involuntary unemployment reduce lifetime income and utility, thereby lowering the long-term benefits of safety measures and increasing the optimal mortality level.

Even higher mortality may result if safety measures are not imposed centrally but determined at the firm-level. First, private agents may not take into account the full social costs of occupational risk via reduced aggregate labor supply. We observe that whether or not this is the case crucially depends on the structure of bargaining and the enforceability of contracts. Second, even if the labor supply externality is internalized, the mortality rate is still affected by the matching externalities common to the DMP framework. Any deviation from the Hosios (1990) condition is found to increase mortality. From a policy perspective, we discuss how taxes can be used to internalize the externalities and show how to design tax schemes that increase occupational safety while keeping the potentially resulting loss in aggregate output at

[1] This is what Freeman and Medoff (1984) coined the "collective voice" function of unions. Robinson (1990) tests the effect of self-reported hazard exposure on quits and prounion sentiment. Although both responses are statistically significant, the estimated marginal effect on the sentiment toward unions is both quantitatively stronger and more precisely estimated.

[2] Empirically, unions have been associated with a lower incidence of fatal work accidents, while their impact on non-fatal work accidents is less clear (Donado, 2015).

a minimum.

The paper proceeds as follows. Section 2 solves the planner's problem for the socially optimal mortality rate in a frictionless labor market. Section 3 introduces the search frictions and solves the planner's problem once again, before turning to the decentralized economy in Section 4. Section 5 concludes. All mathematical proofs are delegated to the appendix.

2 Frictionless labor market

To assess the impact of search frictions on mortality, we first solve the social planner's problem in a frictionless labor market. Each period, the planner can freely allocate the mass N of living individuals between employment and unemployment. The mass of employed and unemployed is denoted by L and U, respectively. While unemployed die at an exogenous rate m_U, the mortality rate of employed, m, is endogenously chosen by the planner. Assuming an exogenous mass of newborns B, the population size evolves according to[3]

$$\dot{N} = B - mL - m_U U. \tag{1}$$

Every unemployed generates a home production of $z > 0$ per period. The production of an employed individual is measured in terms of *effective* output $y(m)$, which captures output minus the costs of safety measures. These costs can be explicit, like regular maintenance of machines or purchasing safety equipment, as well as implicit through lower productivity due to shorter work shifts or time spent on safety routines. The properties of the effective output function are summarized in Assumption 1.

Assumption 1. *For $m \geq 0$, effective output $y(m)$ is twice continuously differentiable and satisfies*

(i) monotonicity and concavity, $y'(m) > 0$, $y''(m) < 0$, with $\lim_{m\to\infty} y'(m) = 0$,

(ii) in present discounted value terms, individuals can produce more with than without a job, $\exists m > 0$ such that $\frac{y(m)}{r+m} > \frac{z}{r+m_U}$,

(iii) but this is not the case at $m = 0$, $\frac{y(0)}{r} \leq \frac{z}{r+m_U}$.

By property (i), the current effective output of an employment relation can be increased by allowing higher mortality as this reduces prevention costs. Concavity implies that these output gains become smaller with increasing mortality. Equivalently, reducing mortality becomes more and more costly the lower it already is. This reflects that an initial drop in mortality can be achieved by relatively cheap measures such as buying safety gloves or glasses, while further reductions in mortality require increasingly expensive measures.

Property (ii) and (iii) are technical. Essentially, property (ii) guarantees that employment is positive in optimum. Property (iii) ensures that the mortality rate of employed individuals

[3] We generally omit time indices to simplify notation. We do not model individual mortality as a state variable in order to keep the model tractable.

is strictly positive, as reducing the mortality rate to 0 would be too costly to make market production preferable over home production.

2.1 Social planner solution

Assuming that all agents have linear utility, the planner's objective is to find time paths of (m, U, L) that maximize the present discounted value of aggregate output,

$$\int_0^\infty [y(m(t))L(t) + zU(t)]e^{-rt}\,dt,$$

subject to the dynamics of the aggregate population (1) as well as $L + U = N$ and $U \in [0, N]$. Ignoring the constraint on U for the moment and substituting $L = N - U$, the current value Hamiltonitian of the planner's problem reads

$$\mathcal{H} = y(m)(N - U) + zU + \nu[B - m(N - U) - m_U U],$$

where ν is the costate to N. Assuming $U < N$, the first order condition with respect to m is

$$\frac{\partial \mathcal{H}}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = \nu. \tag{2}$$

By condition (2), the optimal mortality rate equates the marginal gain of mortality in terms of output, $y'(m)$, to the marginal cost of mortality, which equals the economic value of a life lost, ν. In an optimum, the latter variable evolves over time according to

$$\frac{\partial \mathcal{H}}{\partial N} = -\dot{\nu} + r\nu \quad \Leftrightarrow \quad \dot{\nu} = (r + m)\nu - y(m). \tag{3}$$

From this point onwards we focus on stationary solutions, $\dot{m} = 0$, which by (2) implies $\dot{\nu} = 0$ and reduces (3) to

$$\nu = \frac{y(m)}{r + m}. \tag{4}$$

Hence the value of a life lost equals the present discounted value of foregone production. Combining this with (2), the optimal mortality rate solves

$$y'(m) = \frac{y(m)}{r + m}. \tag{5}$$

Proposition 1 establishes uniqueness of the planner's solution and verifies that the associated optimal level of unemployment is zero. Correspondingly, the population size is $N = L = \frac{B}{m}$ in steady state.[4]

[4]Full employment is clearly due to our agents being *ex ante* homogeneous. If Assumption 1(ii) applied only to a fraction of the individuals (e.g. due to heterogeneity in z or m_U), the model would feature voluntary unemployment. While this is not the point of the paper, it is interesting to note from (5) that as long as the production function is homogeneous, any employed individual faces the same mortality rate.

Proposition 1. *Without search frictions, the social planner's problem has a unique stationary solution with $U^{**} = 0$ and mortality rate $m^{**} > 0$ characterized by* (5)*.*

Condition (5) reveals that the mortality rate m^{**} depends on the discount rate r as well as on the effective production function. The higher r, the less the planner values the future output gains relative to the current output costs of occupational safety, and the higher is the optimal mortality rate. To illustrate the dependence on the shape of the production function, assume $y(m) = Am^{\alpha}$ with $A > 0$ and $\alpha \in (0, 1)$. It is easy to verify $m^{**} = \frac{\alpha}{1-\alpha}r$. Hence the tighter the link between mortality and effective output, the higher is the optimal mortality rate. For $\alpha \to 0$, a reduction in mortality has no detrimental effect on output and thus $m^{**} \to 0$. For $\alpha \to 1$, reducing mortality becomes increasingly costly and $m^{**} \to \infty$.[5]

3 Frictional labor market

3.1 Labor flows

From now on assume that the labor market dynamics are subject to the search and matching frictions typical in the DMP framework. Each period, the mass of unemployed U and the mass of vacancies V are brought together by a constant returns to scale matching function $M(U, V)$. The rate at which vacancies are filled is denoted by $q(\theta) := \frac{M(U,V)}{V} = M(\frac{1}{\theta}, 1)$ where $\theta := \frac{V}{U}$ is the labor market tightness. The rate at which unemployed find a job is $p(\theta) := \frac{M(U,V)}{U} = q(\theta)\theta$. These rates satisfy the standard properties of Assumption 2.

Assumption 2. *The job-finding rate $p(\theta)$ and the vacancy-filling rate $q(\theta)$ are continuously differentiable with*

(i) $\lim_{\theta \to 0} p(\theta) = 0$, $\lim_{\theta \to \infty} p(\theta) = \infty$, $p'(\theta) > 0$,

(ii) $\lim_{\theta \to 0} q(\theta) = \infty$, $\lim_{\theta \to \infty} q(\theta) = 0$, $q'(\theta) < 0$,

(iii) the elasticity $\eta(\theta) := -\frac{q'(\theta)\theta}{q(\theta)}$ *is non-decreasing.*

Everybody is assumed to participate in the labor market, such that $N = L + U$. The population dynamics are governed by the differential equations

$$\dot{L} = -(m + s)L + p(\theta)U, \tag{6}$$

$$\dot{U} = B + sL - (p(\theta) + m_U)U, \tag{7}$$

$$\dot{N} = B - mL - m_U U. \tag{8}$$

[5] Note that the model does not take a stance whether the mortality of employed exceeds the mortality of unemployed. Depending on the parameterization, both outcomes can be achieved. Empirically, mortality rates of unemployed have been found to be higher than those of employed workers in most occupations (Paglione et al., 2020). This is related to increased psychological distress after job loss (Bloemen et al., 2018; Junna et al., 2020), which has been shown to have long-lasting effects on individual mortality (Sullivan and Von Wachter, 2009; Browning and Heinesen, 2012).

The dynamics of the aggregate population (8) are as above. The evolution of the mass of employed and unemployed are described by (6) and (7), respectively. Each period, unemployed find a job at rate $p(\theta)$, while employed move into unemployment at an exogenous rate s. As before, employed individuals die at rate m, while unemployed individuals face an exogenous mortality rate m_U. Newborns start their economic lives without a job.

In a stationary economy with constant inflows, $\dot{B} = 0$, equations (6)–(8) yield

$$L = \frac{p(\theta)}{p(\theta)m + m_U(m+s)}B, \quad U = \frac{m+s}{p(\theta)m + m_U(m+s)}B, \quad N = \frac{m+s+p(\theta)}{p(\theta)m + m_U(m+s)}B.$$

The steady state unemployment rate is $\frac{U}{N} = \frac{m+s}{m+s+p(\theta)}$.

3.2 Social planner solution

If the planner is not bound by the matching frictions and can freely move individuals between employment and unemployment, the analysis is as in Section 2.1. The typical assumption in the matching literature, however, is that the planner cannot overcome the matching frictions and must work through the matching function (Pissarides, 2000, Ch. 8). In contrast to Section 2, the planner cannot control U directly but only indirectly via creating vacancies V. Assuming a flow cost $c > 0$ per vacancy, the planner maximizes

$$\int_0^\infty [y(m(t))L(t) + zU(t) - cV(t)]e^{-rt}\,dt$$

subject to the population dynamics (6)–(8) as well as $L + U = N$ and $U \in [0, N]$. While the planner essentially chooses time paths for (m, V), it is convenient to reformulate the problem in terms of (m, θ) by writing $V = \theta U$. Furthermore, we substitute $L = N - U$ and omit (6) as well as the static constraint on U from the maximization problem. The current value Hamiltonian then reads

$$\mathcal{H} = y(m)(N-U) + zU - c\theta U + \mu[B + s(N-U) - (p(\theta) + m_U)U] + \nu[B - m(N-U) - m_U U],$$

where μ and ν are the costates to U and N, respectively. Assuming $0 < U < N$, the first order conditions for an interior optimum read

$$\frac{\partial \mathcal{H}}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = \nu, \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial \theta} = 0 \quad \Leftrightarrow \quad c = -p'(\theta)\mu. \tag{10}$$

Condition (9) coincides with (2), while condition (10) balances the costs of an additional vacancy with the benefits of lower unemployment.

In an optimum, the dynamics of the costate variables are

$$\frac{\partial \mathcal{H}}{\partial U} = -\dot{\mu} + r\mu \quad \Leftrightarrow \quad \dot{\mu} = (r + s + m_U + p(\theta))\mu + y(m) - z + c\theta - \nu(m - m_U), \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial N} = -\dot{\nu} + r\nu \quad \Leftrightarrow \quad \dot{\nu} = (r + m)\nu - y(m) - s\mu. \tag{12}$$

From this point onwards we again focus on stationary solutions, $\dot{m} = \dot{\theta} = 0$. By the first order conditions, this implies $\dot{\nu} = \dot{\mu} = 0$. Equation (12) gives the economic value of a life lost as

$$\nu = \frac{y(m) + s\mu}{r + m}. \tag{13}$$

Similar to (4), ν equals the present discounted value of a worker's forgone production in case of death. Yet, it now takes into account that the worker may have become unemployed in the future due to a separation shock. Combining (9) and (13) yields

$$y'(m) = \frac{y(m) + s\mu}{r + m}. \tag{14}$$

Comparing this condition to (5) reveals that the search frictions lower the marginal cost of mortality since unemployment reduces a worker's lifetime production in present discounted value terms ($\mu < 0$ by (10)). As a result, search frictions increase the optimal mortality rate, see Section 3.3 for a more thorough analysis.

Substituting (10) and (13) into (11) to replace c and ν, the steady state value of an additional unemployed is

$$\mu = -\frac{(r + m_U)y(m) - (r + m)z}{[r + m_U + p(\theta)\eta(\theta)](r + m) + s(r + m_U)}, \tag{15}$$

where $\eta(\theta) := -\frac{q'(\theta)\theta}{q(\theta)}$ is the elasticity of the vacancy-filling rate. The value of μ corresponds to the change in the present discounted value of output if a worker switches from employment to unemployment. In optimum, this is negative by (10), such that frictional unemployment lowers aggregate output.

Substituting (15) back into (10) yields

$$(1 - \eta(\theta))\frac{(r + m_U)y(m) - (r + m)z}{[r + m_U + p(\theta)\eta(\theta)](r + m) + s(r + m_U)} = \frac{c}{q(\theta)}. \tag{16}$$

Like in the basic DMP model, this equation determines optimal job creation. To pin down the optimal mortality rate, use (15) to eliminate μ from (14), which after some algebra yields

$$y'(m) = \frac{[r + m_U + p(\theta)\eta(\theta)]y(m) + sz}{[r + m_U + p(\theta)\eta(\theta)](r + m) + s(r + m_U)}. \tag{17}$$

With search frictions, a solution to the planner's problem satisfies (16)–(17).

While not immediately visible, it is important to note that (17) is equivalent to $\frac{\partial \mu}{\partial m} = 0$

with μ given in (15). Intuitively, the optimal mortality rate minimizes μ, such that a planner faced with search frictions chooses mortality to make the output loss resulting from frictional unemployment as small as possible. This observation is key to our proof of existence and uniqueness of a solution, which solely focuses on the planner's *job creation curve* $\theta^*(m)$ defined by (16). By Lemma 2 in the appendix, this curve is hump-shaped, which reflects that the planner creates fewer vacancies if the mortality of employed workers is very high (as the expected duration of a production relation is short) but also if mortality is very low (as the required safety measures depress effective output). Since $\mu = -\frac{c}{p'(\theta)}$ by (10), μ is minimized when the tightness θ is maximized. Hence the planner's solution corresponds to the unique peak of the job creation curve as postulated by Proposition 2.

Proposition 2. *With search frictions, the social planner's solution* (m^*, θ^*) *is unique and corresponds to the maximum of the job creation curve* $\theta^*(m)$ *defined by* (16).

This result is graphically illustrated in Figure 1, where JC corresponds to the hump-shaped job creation curve defined by (16). The job destruction curve JD is defined by (17) and downwards sloping. Intuitively, a higher tightness θ increases the job-finding rate and thus μ as the expected output lost in case of unemployment decreases. By (13), this increases the valuation of a worker, ν, and thus the marginal cost of mortality. Therefore, the optimal mortality rate is decreasing in θ along JD. The planner's optimum lies at the intersection of the two curves, which by Proposition 2 coincides with the peak of JC.

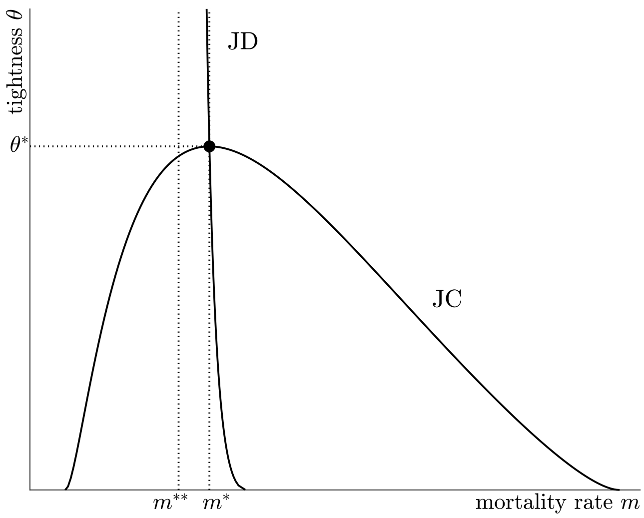

Figure 1. Planner's solution in the presence of search frictions

3.3 The impact of search frictions on mortality

Since the job-finding rate $p(\theta)$ enters the right-hand side of (17), the presence of labor market frictions affects the optimal mortality rate. Hence, in the presence of search frictions, the solution to the planner's problem m^* is only *constrained efficient*.

The difference in the optimality conditions for mortality (5) and (17) essentially arises from the altered value of ν, which measures the value of a life lost in terms of foregone output. Since workers are more productive in jobs than at home ($\mu < 0$), the presence of frictional unemployment decreases a worker's expected lifetime production and thus decreases the marginal costs of mortality. This implies that the optimal mortality rate is higher in the presence of labor market frictions. Proposition 3 shows that this excess mortality increases with the severity of the frictions.

Proposition 3. *Let* $\phi := \frac{s}{r+m_U+p(\theta)\eta(\theta)}$. *The constrained efficient mortality rate* m^* *given in* (17) *is strictly increasing in* ϕ. *For* $\phi \to 0$, *the frictionless mortality rate* m^{**} *given in* (5) *is attained.*

It is straightforward to see from (17) that the optimal mortality rate depends on the labor market frictions only via $\phi = \frac{s}{r+m_U+p(\theta)\eta(\theta)}$. For $s \to 0$ or $\theta \to \infty$, this fraction approaches zero and the frictionless mortality rate is attained. *Ceteris paribus*, the excess mortality caused by the frictions is higher the higher the separation rate and the lower the job-finding rate or the elasticity of the vacancy-filling rate. The fact that $m^* > m^{**}$ is also evident from Figure 1. Proposition 3 implies that the job destruction curve JD approaches the vertical line $m = m^{**}$ for $\theta \to \infty$ and lies to the right of m^{**} for any finite θ. The intersection with the JC curve must thus necessary lie above m^{**}.

By Proposition 3, reforms that accelerate the matching of unemployed to jobs reduce the mortality rate of employed through higher occupational safety. Although not captured by our model, such a reform may at the same time reduce the mortality rate of unemployed m_U due to lower risk of long-term unemployment. Indeed, empirical evidence suggests a positive link between high local unemployment and mortality rates after job loss (Browning and Heinesen, 2012). It is straightforward to verify that the marginal costs of mortality on the right-hand side of (17) are decreasing m_U, such that this channel amplifies the negative effect of search frictions on the provision of occupational safety. Intuitively, a worker's expected lifetime production not only drops due to the presence of unemployment spells, but also because the mortality experienced during unemployed depends on the expected length of these spells.

4 Frictional labor market and bargaining

Having understood the planner's incentives with and without search frictions, we now decentralize the economy studied in the previous section. Mortality is no longer centrally mandated but negotiated by firms and workers together with wages. The attained labor market equilibrium may differ from the planner's solution (m^*, θ^*) due to a range of externalities that are present in the model.

The classical matching externalities may lead the equilibrium tightness to deviate from θ^* (Pissarides, 2000, Ch. 8). This results from the fact that private agents do not take into account that opening an additional vacancy lowers the vacancy-filling probability of all firms, while on the workers' side an additional job-seeker reduces the job-finding probability for all other job-seekers. Additionally, our model features an externality that directly affects the mortality rate. As safety measures are bilaterally negotiated between a firm and a worker, the fact that a worker's death not only terminates the current employment relation but permanently lowers the production capacity of the economy is in general not taken into account.

4.1 Value functions

Each firm consists of one job that can either be filled or vacant. Assuming stationarity, the value of a filled and vacant job are, respectively,

$$
\begin{aligned}
rJ &= y(m) - w - (s+m)(J-V), \\
rV &= -c + q(\theta)(J-V).
\end{aligned}
$$

A filled job generates a flow profit of $y(m) - w$ and is destroyed by an exogenous separation at rate s and by death of the worker at rate m. Assuming free market entry of firms, the value of a vacancy is zero in equlibrium, $V = 0$, implying $J = \frac{c}{q(\theta)}$.

The value of employment and unemployment for the worker are, respectively,

$$
\begin{aligned}
rW &= w - s(W-U) - mW, \\
rU &= z + p(\theta)(W-U) - m_U U.
\end{aligned}
$$

An employed worker consumes her wage, moves to unemployment at rate s and dies at rate m. Unemployed workers consume their home production, find a job at rate $p(\theta)$ and die at rate m_U. The value of death is set to zero since the individual's consumption permanently drops to zero.

4.2 Bargaining

Each period, firm and worker choose a wage w and a mortality rate m that jointly maximize the generalized Nash product

$$
\Psi = (W-U)^{\gamma} J^{1-\gamma}
$$

where $\gamma \in (0,1)$ is the bargaining power of the worker.[6] From above, observe $J = \frac{y(m)-w}{r+m+s}$ and $W = \frac{w+sU}{r+m+s}$. The value of unemployment U is an equilibrium object and taken as given in the bargaining process.

[6]The view that workers and firms bargain over a compensation package that includes non-wage components, has, for instance, been adopted in Dey and Flinn (2005), where the worker's coverage by health insurance is negotiated together with the wage. We study the effect of alternative determination schemes for wages and occupational safety in Section 4.6.

Assuming $W > U$ and $J > 0$, the first order conditions are

$$\frac{\partial \Psi}{\partial w} = 0 \quad \Leftrightarrow \quad \gamma J = (1-\gamma)(W-U), \tag{18}$$

$$\frac{\partial \Psi}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = J + \frac{\gamma J}{(1-\gamma)(W-U)} W. \tag{19}$$

Condition (18) gives rise to the familiar Nash sharing rule, $W - U = \gamma S$ and $J = (1-\gamma)S$ where $S = J + W - U = \frac{y(m)-(r+m)U}{r+m+s}$ is the joint surplus of the match. Substituting this into (19) yields

$$y'(m) = J + W = \frac{y(m) + sU}{r+m+s}. \tag{20}$$

Similarly to the planner's conditions, the left-hand side of (20) measures the marginal benefit of higher mortality in terms of additional output. The right-hand side captures the marginal cost of higher mortality, which in the decentralized economy amounts to losing the match value $J+W$. This value comprises the expected output generated on the current job, $\frac{y(m)}{r+m+s}$, and (via U) the expected income earned on future jobs and during unemployment spells. The negotiating parties internalize the labor supply externality if and only if U is such that (20) coincides with (17), compare Section 4.4.

Notice that the bargaining outcome can be interpreted sequentially. Anticipating that each party will receive a fixed share of the joint surplus, m is chosen to maximize S. This is evident from (20) being equivalent to $\frac{\partial S}{\partial m} = 0$, which will be central to the analysis of the equilibrium below.[7]

4.3 Equilibrium

By the Nash sharing rule, the equilibrium value of unemployment satisfies $(r + m_U)U = z + p(\theta)\gamma S$. Substituting this into the definition of S gives equilibrium surplus

$$S = \frac{(r+m_U)y(m) - (r+m)z}{(r+m_U+p(\theta)\gamma)(r+m) + (r+m_U)s}. \tag{21}$$

We note for further reference that the equilibrium value of unemployment equals

$$U = \frac{p(\theta)\gamma y(m) + (r+m+s)z}{(r+m_U+p(\theta)\gamma)(r+m) + (r+m_U)s}. \tag{22}$$

Plugging (21) into the free entry condition, noting $J = (1-\gamma)S$, yields

$$(1-\gamma)\frac{(r+m_U)y(m) - (r+m)z}{(r+m_U+p(\theta)\gamma)(r+m) + (r+m_U)s} = \frac{c}{q(\theta)}, \tag{23}$$

[7] Formally, $\max_{(w,m)} (W-U)^\gamma J^{1-\gamma} = \max_m \left\{\max_w (W-U)^\gamma J^{1-\gamma}\right\} = \gamma^\gamma (1-\gamma)^{1-\gamma} \max_m S$. In Section 4.6 we discuss the generality of this result.

while substituting (22) into (20) gives, after some algebra,

$$y'(m) = \frac{(r + m_U + p(\theta)\gamma)y(m) + sz}{(r + m_U + p(\theta)\gamma)(r + m) + (r + m_U)s}. \tag{24}$$

A labor market equilibrium $(\hat{m}, \hat{\theta})$ is characterized by equations (23)–(24). Similar to the planner's solution in Section 3.2, it can be verified that the labor market equilibrium corresponds to the peak of the *job creation curve* $\hat{\theta}(m)$, which is now defined by (23). This hinges on the fact that $\hat{m}$ maximizes (21) for $\theta = \hat{\theta}$. Imposing the free entry condition, maximizing (21) is equivalent to maximizing θ along the job creation curve since $S = \frac{c}{(1-\gamma)q(\theta)}$.

Proposition 4. *The equilibrium $(\hat{m}, \hat{\theta})$ is unique and corresponds to the maximum of the job creation curve $\hat{\theta}(m)$ defined by* (23).

The equilibrium looks qualitatively identical to the planner's solution in Figure 1. To gain further economic insights, let us conduct a small comparative static analysis of the equilibrium with respect to the main model variables. Increasing the slope of y around $\hat{m}$ increases joint surplus (21) and hence $\hat{\theta}(m)$ for $m > \hat{m}$ and results in a higher equilibrium mortality rate. The same happens if z or $p(\theta)$ are lowered (for all θ), since surplus decreases relatively more for large m. Thus, *ceteris paribus*, higher mortality rates should be observed in jobs in which prevention measures are most costly and for workers whose outside options are poorest. As shown in Section 4.4.2, the relationship between equilibrium mortality and the bargaining power γ is not monotonic.

4.4 The impact of externalities on mortality

4.4.1 The labor supply externality

The fact that a diseased worker reduces aggregate labor supply is internalized in the firm-level negotiations if the private costs of mortality equal the social costs of mortality. In this case, conditions (14) and (20) coincide, which proofs equivalent to

$$U = \frac{y(m) + (r + m + s)\mu(m, \theta)}{r + m} \tag{25}$$

where $\mu(m, \theta)$ is the shadow price that a planner assigns to an additional unemployed for given (m, θ). This shadow price can be obtained from (11)–(12) and equals

$$\mu(m, \theta) = -\frac{(r + m_U)y(m) - (r + m)(z - c\theta)}{(r + m_U + p(\theta))(r + m) + s(r + m_U)}. \tag{26}$$

Next, note that free entry and Nash bargaining imply $c\theta = \theta q(\theta)(1-\gamma)S = (1-\gamma)p(\theta)\frac{y-(r+m)U}{r+m+s}$ for any U. Substituting this into (26) and plugging the resulting expression into (25) after some algebra yields (22). Therefore, the labor supply externality is internalized in any labor market equilibrium. Even though private agents do not explicitly take into account that a dead worker

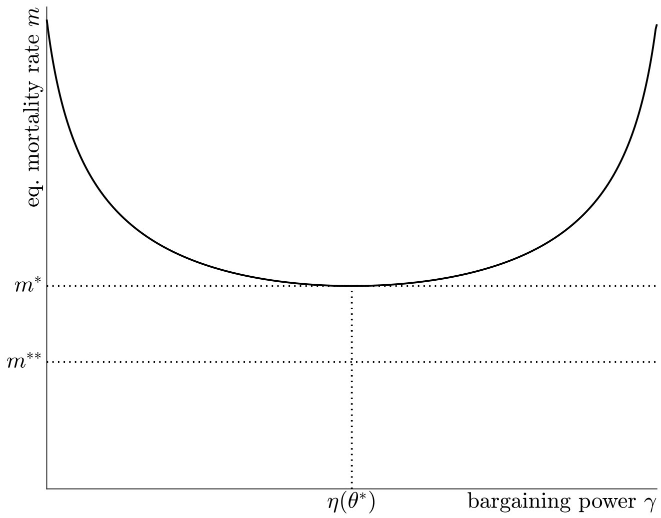

Figure 2. Equilibrium mortality as a function of γ

reduces labor supply on aggregate, in equilibrium this is accurately reflected in the worker's outside option considered in bargaining.

The observation that the labor supply externality is internalized in equilibrium hinges on a particular property of the bargaining scheme studied in Section 4.2. As mentioned there, the bargained mortality rate maximizes the *joint* surplus of firms and workers. This is essential, since the production potential outside the firm is only taken into account by the worker, but not by the firm. In Section 4.6 we investigate alternative schemes to determine safety levels and their ability to internalize the labor supply externality.

4.4.2 Matching externalities

Even though the labor supply externality is internalized, the equilibrium mortality rate need not be efficient. It may still be distorted by the presence of externalities that arise from the matching process and affect the equilibrium value of U in (20). Indeed, we observe that the equilibrium conditions (23)–(24) coincide with the planner's conditions (16)–(17) if and only if $\gamma = \eta(\theta)$, which corresponds to the familiar Hosios (1990) condition. In this case, the labor market equilibrium is constrained efficient and attains the mortality rate m^*.

Even if the Hosios condition holds, mortality is higher than in a frictionless labor market. Therefore, one may ask whether a deviation from the Hosios condition may lead to lower mortality rates. Proposition 5 negates this. It shows that *any* deviation from the Hosios condition increases the mortality rate above its constrained efficient level m^*. Hence both a too low and a too high degree of worker representation in wage negotiations aggravates the negative effects of search frictions on mortality.

Proposition 5. *The equilibrium attains the constrained efficient mortality rate m^* if and only if $\gamma = \eta(\theta^*)$. Otherwise, the equilibrium mortality rate exceeds m^*.*

The result of Proposition 5 is illustrated in Figure 2. The relation between bargaining power and equilibrium mortality is U shaped. This reveals that unionization can help to move the economy closer to efficient provision of occupational safety only if worker's individual bargaining power is sufficiently low. At the same time, however, the bargaining power of the union should not be too high since the mortality rate starts to increase in γ at some point. This is because high negotiated wages reduce worker's job finding rates, which reinforces the search frictions and increases mortality.

The turning point occurs at $\gamma = \eta(\theta^*)$, i.e. when the Hosios condition is satisfied. This is because the equilibrium mortality rate is inversely related to the equilibrium value of unemployment U. Lemma 3 in the appendix shows that U peaks at $\gamma = \eta(\theta^*)$ such that mortality achieves its minimum at this point, where it attains the constrained efficient rate m^*.[8] Even if all externalities are internalized, the mortality rate is still higher than in a frictionless labor market, where it equals m^{**}. While appropriately designed policies can reduce mortality below m^*, this comes with a loss in aggregate output as discussed in the next section.

4.5 Policy

Suppose that there is a government who, while maximizing output, wants to keep the equilibrium mortality rate below some $\bar{m}$. For $\bar{m} \geq m^*$, it is clear from Section 3.2 that the desired pair is the planner's solution (m^*, θ^*). By Proposition 5 this is attained as equilibrium if the Hosios condition is satisfied, such that the government's efforts should concentrate on enforcing this condition. If the bargaining power of the negotiating parties cannot directly be adjusted, a transfer between workers and firms may implement to the same labor market outcomes. We return to this towards the end of this section.

For now, let us assume that the matching elasticity is constant $\eta(\theta) \equiv \eta$ and that the Hosios condition holds, $\gamma = \eta$. As argued above, no policy intervention is necessary if $\bar{m} \geq m^*$. Otherwise, we know from the analysis of Section 3.2 that the optimal mortality rate is $\bar{m}$ and the associated tightness $\bar{\theta}$ lies on the planner's job creation curve (16) illustrated by the solid line in Figure 3.

We now seek for policies that decentralize $(\bar{m}, \bar{\theta})$ as an equilibrium. To this purpose, we consider a mortality-dependent tax on firms, which changes effective output from $y(m)$ to $y(m) - \Delta(m)$. In equilibrium, all tax revenue is equally distributed among all individuals by a lump sum transfer t. With this policy, the job creation curve in the decentralized economy (23) becomes

$$(1-\eta)\frac{(r+m_U)(y(m)-\Delta(m)+t)-(r+m)(z+t)}{(r+m_U+p(\theta)\eta)(r+m)+(r+m_U)s} = \frac{c}{q(\theta)}.$$

For the equilibrium to lie on the planner's job creation curve (16), the terms arising from the

[8] The property that the equilibrium value of unemployment is maximized under the Hosios condition is inherited from the basic DMP model, see Pissarides (2000, p.187). Endogenous mortality does not destroy this property since $\frac{\partial U}{\partial m} = \frac{\partial S}{\partial m} = 0$ for any given γ.

policy must cancel, i.e. $(r + m_U)\Delta(\bar{m}) = (m_U - \bar{m})t$. Additionally, a balanced budget requires

$$\Delta(\bar{m})L = tN \quad \Leftrightarrow \quad p(\bar{\theta})\Delta(\bar{m}) = (m + s + p(\bar{\theta}))t.$$

Combining the two equations reveals $t = \Delta(\bar{m}) = 0$, such that in equilibrium the size of the intervention should be zero. Furthermore, the policy changes equation (24) to

$$y'(m) - \Delta'(m) = \frac{(r + m_U + p(\theta)\eta)(y(m) - \Delta(m) + t) + s(z + t)}{(r + m_U + p(\theta)\eta)(r + m) + (r + m_U)s}.$$

Evaluating this in equilibrium, using $t = \Delta(\bar{m}) = 0$, yields

$$\Delta'(\bar{m}) = y'(\bar{m}) - \frac{(r + m_U + p(\bar{\theta})\eta)y(\bar{m}) + sz}{(r + m_U + p(\bar{\theta})\eta)(r + \bar{m}) + (r + m_U)s}. \tag{27}$$

For $\bar{m} < m^*$ the right-hand side of (27) is positive, such that $\Delta'(\bar{m}) > 0$. Hence although the tax is zero in equilibrium, the tax schedule is upwards sloping, which increases the marginal cost of mortality. The gradient of the tax schedule must be such that marginal costs and marginal benefits of higher m are equalized at $\bar{m}$.

Note that the above conditions only pin down $\Delta(\bar{m})$ and $\Delta'(\bar{m})$, but not the function values in other points. The specific shape of Δ in fact does not matter as long as no additional equilibrium arises. This is granted if the altered effective output function $y(m) - \Delta(m)$ satisfies Assumption 1. One such tax schedule is

$$\Delta(m) = \lambda[y(m) - y(\bar{m})],$$

with which the government captures a share λ of the production gain that arises from producing with a mortality rate above the target. By construction, $\Delta(\bar{m}) = 0$, while $\Delta'(\bar{m}) = \lambda y'(\bar{m})$. Substituting this into (27) pins down λ as

$$\lambda = 1 - \frac{(r + m_U + p(\bar{\theta})\eta)y(\bar{m}) + sz}{[(r + m_U + p(\bar{\theta})\eta)(r + \bar{m}) + (r + m_U)s]y'(\bar{m})}.$$

The resulting job creation curve is illustrated by the dashed line JC' in Figure 3. To implement $(\bar{m}, \bar{\theta})$ as an equilibrium, the policy must be such that the job creation curve of the decentralized economy is maximized in this point, compare Proposition 4. While mortality is lower in the new equilibrium, this comes at the cost of lower job creation due to additional safety measures. Since aggregate production is maximized at m^*, the policy leads to a loss in output.

If the Hosios condition is not satisfied, $\gamma \neq \eta$, and the government cannot directly affect the bargaining weight, the tax scheme presented above can be adjusted to take this into account. The required tax is then no longer zero in equilibrium, but accounts for the deviation between γ and η. The tax on firms will be negative (reflecting a transfer) if the workers' bargaining power is too high, $\gamma > \eta$. Otherwise, the tax is positive in equilibrium. Since this intervention alters

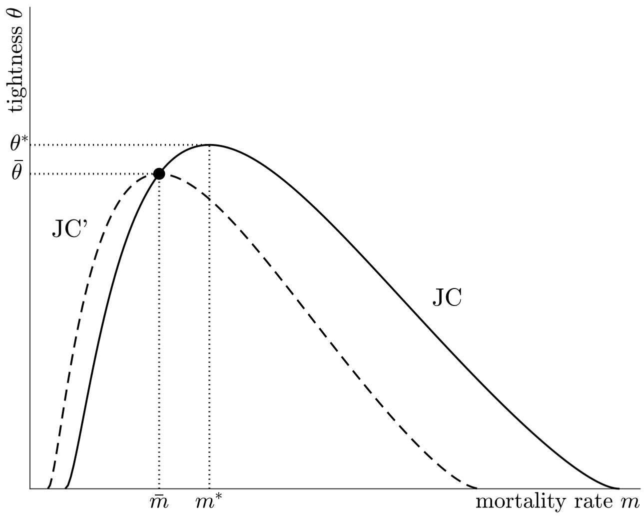

Figure 3. Labor market equilibrium with and without policy

the marginal costs of mortality in equilibrium, the slope of the tax schedule is no longer given by (27), but includes an additional term reflecting $\gamma - \eta$.

4.6 Alternative determination schemes for occupational safety

As demonstrated in Section 4.4.1, the joint bargaining of wages and safety measures as assumed in Section 4.2 leads to the internalization of the labor supply externality in equilibrium. The same is true if occupational safety is determined differently, as long as the outcome maximizes joint surplus.

Consider, for instance, that just the wage is bargained, while m is unilaterally set by the firm *before* the wage negotiation. Since firms anticipate that the joint surplus will be shared according to the Nash rule by (18), they solve

$$\max_m J = (1-\gamma)S \quad \Leftrightarrow \quad \max_m S$$

at the first stage. Hence the equilibrium obtained under this bargaining protocol coincides with the equilibrium of Section 4.3. This also holds if the worker could unilaterally decide on m by maximizing W, or if m would be the result of yet another bargain, irrespective of the bargaining weights.

It is essential, however, that the level of safety measures is determined *before* wages or *simultaneously* with wages. Otherwise, the firm has an incentive to underinvest. To see this, assume that firms can unilaterally choose m *after* wages w have been set. Their optimal choice maximizes J subject to $W \geq U$ for the given wage level w. The first order condition for an interior optimum is

$$y'(m) = J = \frac{y(m) - w}{r + m + s}.$$

The marginal cost of mortality considered is now only the firm's private cost J that results from the termination of the match. The cost of the worker is not taken into account. Comparison with (20) reveals that, irrespective of the wage, mortality is higher than in the equilibrium of Section 4.3.[9]

This observation implies that in the equilibrium of Section 4.3, firms have an incentive to deviate from negotiated levels of m and underinvest into safety measures to increase their profit *ex post*. The equilibrium is thus not feasible if firms lack commitment and negotiation outcomes cannot be enforced by courts. In this case, a strong collective voice as provided by unions or other forms of worker representation may help to keep firms accountable for their promises (Freeman and Medoff, 1984). Alternatively, a tax scheme as suggested in Section 4.5 can align the firm's marginal cost of mortality with the social marginal cost. This eliminates the firm's incentive to deviate *ex post*, implementing the equilibrium of Section 4.3 even if the firm cannot commit to any *ex ante* agreed level of m.[10]

Firms may also underinvest into safety measures when these are set *before* wages if part of their costs are irretrievable in case no agreement is reached in the wage negotiations. This changes a firm's threat point in the bargain as it would incur a loss in case the worker walks away. This is an example of the classical hold-up problem.[11] Assuming sunk costs $d(m) > 0$, the bargaining problem becomes

$$\max_{w} (W - U)^{\gamma} (J + d(m))^{1-\gamma}$$

since the firm's outside option is now $-d(m)$. The solution implies $J = (1 - \gamma)S - \gamma d(m)$. Assuming that the firm unilaterally chooses the level of safety measures *before* wages are negotiated, the first order condition for m is

$$\frac{\partial S}{\partial m} = \frac{\gamma}{1-\gamma} d'(m).$$

If higher safety measures require more upfront costs, $d'(m) < 0$, the firm chooses a point on the downwards sloping part of the surplus curve, $\frac{\partial S}{\partial m} < 0$. Therefore, joint surplus is no longer maximized. The above optimality condition is equivalent to

$$y'(m) - \frac{\gamma}{1-\gamma} d'(m)(r + m + s) = \frac{y(m) + sU}{r + m + s},$$

which shows that the firm's marginal gain of mortality increases because part of the additional expenditures on safety measures cannot be shared with the worker. Even if $d'(m) = 0$, the presence of sunk costs affects the mortality rate via U as the worker effectively receives a higher share in surplus.

[9]This also holds if m is the result of a bargain, provided that m is bargained after w and the worker's bargaining weight is smaller than in the wage negotiation.

[10]To implement the equilibrium $(\hat{m}, \hat{\theta})$ of Section 4.3, the tax must satisfy $\Delta(\hat{m}) = 0$ and $\Delta'(\hat{m}) = W = \gamma S$ with equilibrium surplus S given in (21).

[11]See Malcomson (1997) for a summary of this literature.

Joint determination of wages and safety levels as in Section 4.2 can correct these distortions and avoid hold-up. It is likely, however, that some safety expenditures cannot be postponed for technical reasons. Consider, for example, structural safety facilities that are planned in a new plant well in advance of workers being hired. Similarly, new employees benefit from already existing safety facilities. In such cases, unions could be efficiency enhancing, for example by negotiating a firm-level collective bargaining agreement through which also future workers participate in the costs through lower wages. These positive effects on safety may be offset, however, if collective bargaining at the same time increases workers' bargaining power, which reduces the firm's incentive to invest. Alternatively, a policy along the lines of Section 4.5 can be used to redistribute a part of the firm's upfront expenditures to the households.

While in a bargaining setting, hold-up and matching externalities may lead to suboptimal outcomes, efficiency may arise automatically if the labor market is organized differently. Assume that firms post and credibly commit to contracts (m, w) to which workers apply in the manner of directed search (Moen, 1997; Acemoglu and Shimer, 1999). The equilibrium is characterized as the solution to

$$\max_{(m,w,\theta)} p(\theta)W \quad \text{s.t.} \quad q(\theta)J = c.$$

It is easy to verify that the directed search equilibrium satisfies the planner's conditions (16)–(17) and thus internalizes all externalities. Additionally, directed search reduces the potential for hold-up problems (Acemoglu and Shimer, 1999).

5 Conclusion

This paper studied the provision of occupational safety in a labor market with search frictions. To this purpose, the basic Diamond-Mortensen-Pissarides model was extended for mortality shocks whose probability is determined endogenously. The presence of search frictions was found to increase the socially optimal mortality rate. While the marginal costs of safety measures are unaffected by the frictions, periods of involuntary unemployed decrease a worker's expected lifetime production and hence the long-run gains of safety measures.

In a decentralized setting, a range of externalities may lead to a further increase in mortality. Assuming joint bargaining of wages and safety levels, we found that the negotiating parties internalize the labor supply externality, i.e. the effect of a higher mortality rate on aggregate labor supply. This is far from obvious, since none of the parties explicitly takes the aggregate effects of their decisions into account. Yet, in equilibrium the worker's outside option reflects the correct "price" of mortality. This requires, however, that firms can be held accountable for their promises as they could increase their profit *ex post* by underinvesting into workplace safety.

Even if the labor supply externality is internalized, the Hosios (1990) condition is required to attain the planner's constrained efficient mortality rate. Any deviation from the Hosios condition leads to inefficiently high mortality due to matching externalities. Thus, while unionization can help move the economy closer to efficient provision of occupational safety, too high bargaining power of unions is detrimental.

Public policies that mandate or incentivize firms to invest more into occupational safety can be effective to align private and social costs of mortality. While in our model misguided private incentives only stem from labor supply and matching externalities, many other factors potentially distort the private provision of occupational safety in the real world (Pouliakas and Theodossiou, 2013). The recent policy initiatives in the EU and the US reflect that policymakers acknowledge these problems and are willing to take action.

Such policies, however, are less suitable to address the excessive mortality caused by search frictions. As we demonstrated, once all externalities have been internalized, a further policy-induced reduction in mortality inevitably lowers aggregate economic output and hence welfare. To address the detrimental mortality effects of search frictions, these should be addressed more directly. Accelerating the matching of unemployed to job openings, for example, at the same time increases occupational safety and aggregate output. Along these lines, the recent rise in long-term unemployment after the COVID crisis may inhibit the success of the recent policy initiatives to boost occupational safety if labor market frictions remain elevated.[12]

The model presented in this paper was purposefully kept simple to identify the main mechanisms that affect the provision of occupational safety in a labor market with search frictions. We believe that these mechanisms will remain of central importance in more complex versions of the model. Indeed, our model is general enough to be extended in many directions. For instance, premature death of a worker is arguably the most extreme implication of low occupational safety. A lot of adverse economic effects already occur during the worker's lifetime in the form of health deficits that lead to chronic diseases or permanent disability. In modern welfare states, a big part of health expenditures are born by the public and are thus not reflected in private decision-making. This creates an externality absent in the presented model. Furthermore, we abstracted from modeling individuals' education decisions, which ultimately determines the characteristics of their future potential jobs. Distortions in the provision of occupational safety are likely to distort schooling decisions and occupational choices as well. We also neglected life-cycle features. Individual attitudes towards health hazards may vary over a worker's lifetime depending on age, health, and socioeconomic factors. This may call for policies that focus on particular subpopulations. These and further questions are left for future research.

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[12] Compare the blog entry by Pissarides (2020) on the potential persistent increase in long-term unemployment brought by accelerated automation.

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A Mathematical appendix

This section contains auxiliary results and proofs to the propositions stated in the article.

A.1 Auxiliary results

Lemma 1. *The function* $\phi(m) := \frac{y(m)}{r+m}$ *satisfies* $\lim_{m\to\infty} \phi(m) = 0$. *It is unimodal with a single peak* $\overline{m} > 0$, *which satisfies* $\frac{y(\overline{m})}{r+\overline{m}} > \frac{z}{r+m_U}$.

Proof. Assumption 1(i) implies $\lim_{m\to\infty} \phi(m) = 0$ by L’Hopital’s rule. The derivative is $\phi'(m) = \frac{r}{r+m}[y'(m) - \frac{y(m)}{r+m}]$. At any point that satisfies $\phi'(m) = 0$, the second derivative is $\frac{r}{r+m}y''(m) < 0$. Hence any local optimum of ϕ is a maximum. Assumption 1(ii) guarantees an $\tilde{m} > 0$ such that $\phi(\tilde{m}) > \frac{z}{r+m_U} \geq 0$. As ϕ asymptotically approaches 0, it either has a single peak $\overline{m} > 0$ or is monotonically decreasing. The latter is ruled out by Assumption 1(iii), which implies $\phi(0) < \phi(\tilde{m})$. Finally, $\phi(\overline{m}) \geq \phi(\tilde{m}) > \frac{z}{r+m_U}$, since $\overline{m}$ maximizes ϕ. □

Lemma 2. *Equation* (16) *defines a function* $\theta^*(m)$ *with the following properties:*

(i) the domain of θ *is a non-empty interval* $M = (\underline{m}, \overline{m}) \subset \mathbb{R}^+$ *whose boundaries satisfy* $\frac{y(m)}{r+m} = \frac{z}{r+m_U}$,

(ii) the sign of $\frac{d\theta^*}{dm}$ *is the opposite of* $\frac{\partial \mu}{\partial m}$ *where* μ *is given in* (15),

(iii) the function is unimodal with a single peak and approaches zero at the boundaries of M.

Proof. Property (i): Since $q(\theta)$ is positive for any finite θ by Assumption 2, a solution to (16) can only exist if $\mu < 0$, which requires $m \in M := \{m \geq 0 : \frac{y(m)}{r+m} > \frac{z}{r+m_U}\}$. On the other hand, for any $m \in M$ the properties of Assumption 2 ensure that (23) has a unique solution $\theta^*(m) > 0$. Hence the domain of θ^* is M, which by Assumption 1(iii) does not include zero. The unimodality result of Lemma 1 implies that M is a non-empty open interval.

Property (ii): Applying the implicit function theorem to (16) gives

$$\frac{d\theta^*}{dm} = \frac{-(1-\eta(\theta))\frac{\partial \mu}{\partial m}}{(1-\eta(\theta))\frac{\partial \mu}{\partial \theta} - \eta'(\theta)\mu - \frac{c}{q(\theta)^2}q'(\theta)}$$

with μ given in (15). Since $\mu < 0$, $\frac{\partial \mu}{\partial \theta} > 0$, and Assumption 2, the denominator is strictly positive. Furthermore, $1 - \eta(\theta) = \frac{p'(\theta)\theta}{p(\theta)} > 0$, such that the sign of $\frac{d\theta^*}{dm}$ equals the sign of $-\frac{\partial \mu}{\partial m}$.

Property (iii): At the boundaries of M, $\frac{y(m)}{r+m} \to \frac{z}{r+m_U}$ and thus $\frac{c}{q(\theta)} \to 0$. By Assumption 2(ii), this implies $\theta \to 0$. Since $\theta(m) > 0$ for $m \in M$, θ must attain a local maximum on M. To verify that this maximum is unique, I rule out the existence of inner local minima. Property (ii) of this Lemma implies that $\frac{d\theta^*}{dm} = 0$ if and only if $\frac{\partial \mu}{\partial m} = 0$. Every such point is a local maximum since $\frac{d^2\theta^*}{dm^2}$ becomes proportional to $-\frac{\partial^2 \mu}{dm^2} = \frac{ry''(m)}{(r+m_U+p(\theta)\eta(\theta))(r+m)+s(r+m_U)} < 0$. By continuity, $\frac{d\theta^*}{dm}$ cannot change its sign more than once such that the maximum is unique. □

Lemma 3. *The equilibrium value of unemployed U is unimodal in γ and peaks at $\gamma = \eta(\theta)$.*

Proof. Plugging (21) into $(r + m_U)U = z + p(\theta)\gamma S$ yields

$$U = \frac{p(\theta)\gamma y(m) + (r + m + s)z}{(r + m_U + p(\theta)\gamma)(r + m) + (r + m_U)s}.$$

Differentiation with respect to γ gives

$$\frac{dU}{d\gamma} = \frac{\partial U}{\partial m}\frac{dm}{d\gamma} + \frac{\partial U}{\partial[p(\theta)\gamma]}\frac{d[p(\theta)\gamma]}{d\gamma}.$$

Observe that $\frac{\partial U}{\partial m} = 0$ and $\frac{\partial U}{\partial[p(\theta)\gamma]} = \frac{r+m+s}{(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s}S$. Hence the sign of $\frac{dU}{d\gamma}$ coincides with the sign of $\frac{d[p(\theta)\gamma]}{d\gamma}$, which is shown to equal the sign of $\eta(\theta)-\gamma$ in the proof of Proposition 5. The rest of the proof is analogous to there. □

A.2 Proofs of propositions

Proof of Proposition 1. We first show that (5) defines a unique mortality rate. Note that (5) corresponds to the first order condition of $\max_m \frac{y(m)}{r+m}$. By Lemma 1, this function is unimodal with a single peak, such that there is exactly one m that satisfies this condition.

Next, note that (5) was obtained assuming $U < N$. The maximized value of the Hamiltonian is $\mathcal{H}^{**} = \frac{ry(m^{**})}{r+m^{**}}(N-U) + zU + \frac{y(m^{**})}{r+m^{**}}[B - m_U U]$. To determine the optimal value of U, observe $\frac{\partial \mathcal{H}^{**}}{\partial U} = z - \frac{r+m_U}{r+m^{**}}y(m^{**})$. As m^{**} maximizes $\frac{y(m)}{r+m}$, the derivative is strictly negative by Lemma 1. Therefore, $U^{**} = 0$ and the initial assumption is satisfied. □

Proof of Proposition 2. By Lemma 2, $\theta^*(m)$ has a unique peak that is characterized by $\frac{\partial \mu}{\partial m} = 0$ where

$$\frac{\partial \mu}{\partial m} = -\frac{r + m_U}{(r + m_U + p(\theta)\eta(\theta))(r + m) + rs}\left[y'(m) - \frac{(r + m_U + p(\theta)\eta(\theta))y(m) + sz}{(r + m_U + p(\theta)\eta(\theta))(r + m) + (r + m_U)s}\right].$$

Hence the point (m, θ) that maximizes $\theta^*(m)$ solves the planner's problem since it satisfies (16)–(17). On the other hand, any solution satisfies $\frac{\partial \mu}{\partial m} = 0$ and therefore corresponds to an interior extremum of $\theta^*(m)$. Since $\theta^*(m)$ is unimodal, the only interior extremum is the unique global maximum. □

Proof of Proposition 3. Equation (17) can be rewritten $y'(m) = \frac{y(m)+\phi z}{r+m+\phi(r+m_U)}$. For $\phi = 0$, the condition simplifies to (5). The implicit function theorem implies $\frac{dm}{d\phi} = \frac{\mu}{y''(m)[r+m+\phi(r+m_U)]}$. The derivative is positive since $y''(m) < 0$ and $\mu < 0$. □

Proof of Proposition 4. The result immediately follows from Lemma 2 and Proposition 2 by setting $\eta(\theta) = \gamma$ and noting that $\mu = -S$. □

Proof of Proposition 5. I first verify that like in the basic DMP model, the equilibrium tightness is strictly decreasing in γ. Consider the total derivative of (23),

$$[(1-\gamma)\tfrac{\partial S}{\partial \gamma} - S]d\gamma + [(1-\gamma)\tfrac{\partial S}{\partial \theta} + c\tfrac{q'(\theta)}{q^2(\theta)}]d\theta + (1-\gamma)\tfrac{\partial S}{\partial m}dm = 0,$$

where all expressions are evaluated in equilibrium and S is given in (21). Since m maximizes S, the last term is zero and evaluating the remaining terms yields

$$\frac{d\theta}{d\gamma} = -\frac{(r+m_U+p(\theta))(r+m)+(r+m_U)s}{[p(\theta)\gamma+\eta(\theta)(r+m_U)](r+m)+\eta(\theta)(r+m_U)s} \cdot \frac{\theta}{1-\gamma} < 0.$$

Second, observe from (24) that the equilibrium mortality rate depends on γ only via the joint term $p(\theta)\gamma$. The implicit function theorem reveals

$$\frac{\partial m}{\partial[p(\theta)\gamma]} = \frac{y(m)-y'(m)(r+m)}{y''(m)[(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s]} < 0.$$

The sign follows from $y'' < 0$ and substituting (24) by which $y(m) > y'(m)(r+m)$. Furthermore,

$$\frac{d[p(\theta)\gamma]}{d\gamma} = p(\theta) + p'(\theta)\gamma\frac{d\theta}{d\gamma} = p(\theta)\left[1+(1-\eta(\theta))\frac{d\theta}{d\gamma}\frac{\gamma}{\theta}\right].$$

Substituting $\frac{d\theta}{d\gamma}$ from above and collecting terms yields

$$\frac{d[p(\theta)\gamma]}{d\gamma} = p(\theta)\frac{(r+\gamma p(\theta))(r+m)+rs}{[\gamma p(\theta)+\eta(\theta)r](r+m)+\eta(\theta)rs}\frac{\eta(\theta)-\gamma}{1-\gamma}.$$

Putting things together, the sign of $\frac{dm}{d\gamma} = \frac{\partial m}{\partial[p(\theta)\gamma]}\frac{d[p(\theta)\gamma]}{d\gamma}$ equals the sign of $\gamma - \eta(\theta)$. Since $\frac{d\theta}{d\gamma} < 0$ and $\eta' \geq 0$, it follows that $\gamma - \eta(\theta)$ is strictly increasing in γ. Therefore, m has a unique minimum, which satisfies $\gamma = \eta(\theta)$. □