Strong contextuality by non-faithful emeddability

http://tph.tuwien.ac.at/~svozil/publ/2021-XCQFArgentina2021-pres.pdf

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Preamble

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- Q quantum mechanical entities being formalized by entities of Hilbert space. In particular, pure states are identified with unit vectors, maximal observables aka "contexts" are identified with orthonormal bases.
- C In contradistinction, classical entities are based on Boolean algebras, and classical probabilities are based on convex combinations of "extreme" cases identified with two-valued states on them.
- M Metaphysical conjecture/working hypothesis: Any measurement "creates"—"carves out" an "emergent property" that cannot be classically "pre-existent" relative to the presumtions (eg, non-contextuality).

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    2.2. based on gadget graphs with input/output terminals—aka pre-/postselection of pure quantum states: (Kochen-)Specker bug (1965, aka Hardy-type, cf Stigler's law of eponymy), Belinfante, Stairs, Cabello, ...;

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    3.1. **nonfaithful embedding into Boolean algebras associated with inseparability, nonunital value assignments, and other nonclassical properties;**
    3.2. nonexistence of any classical interpretation aka two-valued (even partial) states: Gleason, Specker, Zierler-Schlessinger, Kamber, Kochen-Specker, Pitowsky, Hrushovski-Pitowsky, Cabello, Abbot-Calude-Svozil ...;

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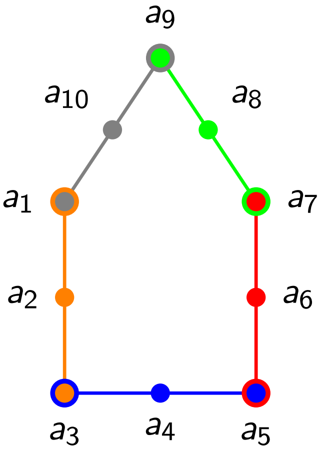
- Choice of the distribution depends on the physical / psychological *etc* realization of the *BOO*.

Anecdotal example: probabilities on a cyclic logic whose respective hypergraph is a pentagon aka pentagram aka house



1) classical probability distributions in terms of convex combinations of the 11 two-valued states thereon;

2) quantum probability distributions according to Born, Gleason, and Lovász;

3) exotic probability according to Gerelle & Greechie & Miller (1974) and Wright (1978)

4) — ... ?

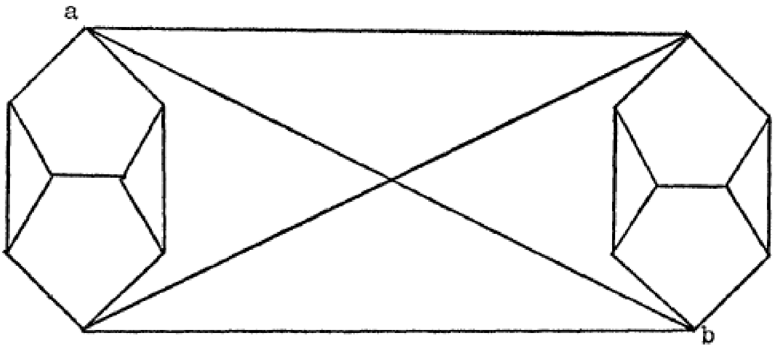
So far we only spoke about comparing
different probability distributions on fixed collections of
(interwined)observables ...

interlude

... now we shall be talking about
"weird" nonclassical collections of (interwined)observables ...

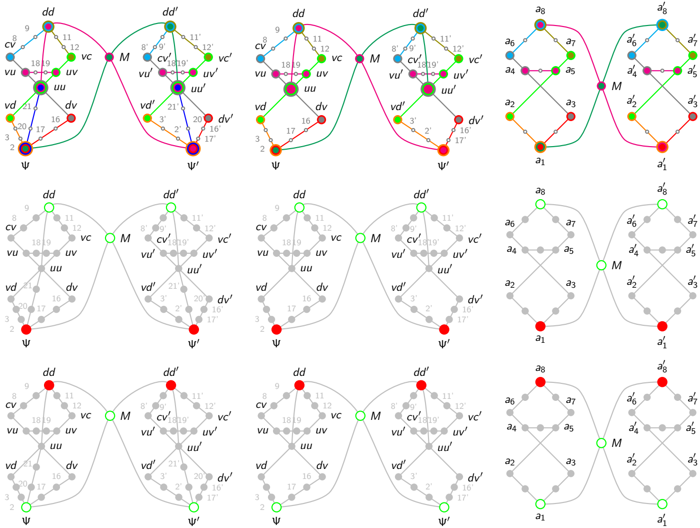
Inseparability 101: Kochen & Specker's demarcation criterion 1967, Theorem 0 of DOI: 10.1512/iumj.1968.17.17004

Theorem 0. *Let $\mathfrak{N}$ be a partial Boolean algebra. A necessary and sufficient condition that $\mathfrak{N}$ is imbeddable in a Boolean algebra B is that for every pair of distinct elements a, b in $\mathfrak{N}$ there is a homomorphism $h: \mathfrak{N} \to Z_2$ such that $h(a) \neq h(b)$.*



Graph of Γ_3

Hypergraphs with nonseparable set of two-valued states third column is Kochen & Specker (1967, Γ_3)



KS, DOI:10.1103/PhysRevA.103.022204

Hypergraph with nonunital set of 6 value assignments

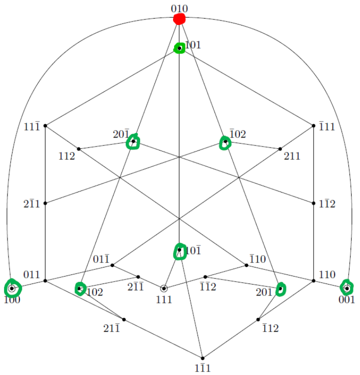
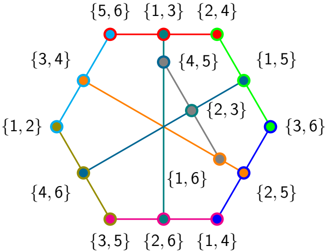


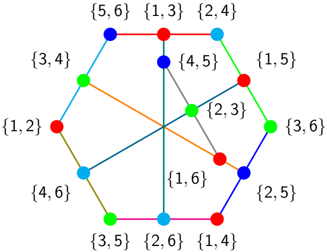
Fig. 2 ‘Almost’ Greechie diagram of a suborthoposet of $L(\mathbf{R}^3)$ without a unital set of two-valued states [e.g., $12\bar{1} = \mathrm{Sp}(1, 2, -1)$].

Josef Tkadlec, DOI:10.1023/A:1026646229896 based on Erna Clavadetscher-Seeberger, Diss. ETH Zürich (Specker) handle ETH: 20.500.11850/138142 based on Schütte's letters to Specker, April 22nd, 1965 & November 3rd, 1983 (communicated to KS by Specker).

Hypergraph with exotic contextuality derived from coloring

Hypergraph of biconnected intertwined contexts representing complete graphs with a separating set of 6 two-valued states which is non-partitionable: G_{32}, cf. Figure 6, p. 121 Greechie (1971) DOI: 10.1016/0097-3165(71)90015-X





Mohammad H. Shekarriz & KS, vertex labeling by partitions of $\{1, 2, 3, 4, 5, 6\}$ with no faithful orthogonal representation arXiv:2105.08520.

Thank you for your attention!