

Global sensitivity analysis of a techno-socio-economic building stock energy model

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Abstract

Building stock models are increasingly applied for deriving policy recommendations in achieving climate neutrality of the building sector. However, they are associated with significant uncertainties, which usually are not systematically analysed. We apply the elementary effects method on scenarios calculated with the model Invert/EE-Lab. Among others, we identified parameters relevant for the decision making algorithm within the model as most influential. This has implications on the interpretation of results, in particular for the future mix of energy carriers which show very similar costs and attractiveness. The systematic application of global sensitivity analyses can help to improve the understanding of building stock modelling results and better substantiate related policy conclusions.

Key Innovations

- This paper presents a global sensitivity analysis of the techno-socio-economic building stock energy model Invert/EE-Lab in scenarios up to 2030 using the Elementary Effects Method as implemented in the SAFE-Toolbox (Pianosi et al, 2015).
- Techno-economic building stock models in general are very input data intensive. Despite this large amount of relevant and by nature uncertain input data, sensitivity analysis are only rarely carried out in a systematized way, partly because of limitations of computational efforts.
- Thus, we consider the application of a global sensitivity analysis for identifying the most sensitive input parameters in an established building stock model carried out for several EU countries in this way as a highly innovative and relevant aspect.
- Besides implementing the general approach and deriving new, innovative conclusions for the further work, also computational wise the work presented in this paper is innovative in our field: In total, more than 20,000 simulation runs per country, partly on a scientific computing cloud service have been carried out, requiring efficient

handling in the processing of input and output data.

Practical Implications

First, we expect that as practical implementation of our work sensitivity analyses will become more common practice in this field of modelling, leading to a better identification of most sensitive input parameters and thus to a better understanding of model results. Second, the conclusions regarding the parameters identified as the most sensitive ones may serve as indication for special attention to these type of parameters in future modelling work. Finally, this should help to improve the policy recommendations derived from building stock models.

Introduction

For various planning and policy issues the scenario development of future heating and cooling demand as well as the related HVAC systems, energy carrier mix and resulting GHG-emissions is of great importance. For this purpose, building stock bottom-up models are more and more frequently used. However, techno-socio-economic building stock energy models used for energy and emission development projections and policy assessment involve considerable uncertainty: The models are very input-data intensive, e.g. data on the existing building stock, in particular in case of high granularity of the archetypes applied in the models, techno-economic data of technologies and measures (e.g. renovation measures, heating systems), future energy prices, interest rates, lifetime of technologies or parameters of decision making algorithms. Each of these input data has a potentially high impact on the results. Partly because of the high computational effort related with sensitivity analyses, systematic uncertainty analysis and sensitivity analyses are rarely applied for building stock models.

The key research question of this paper is: What are the input parameters showing the highest impact on results in techno-socio-economic building stock models for developing scenarios of future energy demand and technology uptake? By elaborating answers to this question for the case of the model Invert/EE-Lab, we analyse and demonstrate the applicability of global sensitivity analyses on data-intensive techno-socio-economic bottom-up building stock models.

There is a broad literature on sensitivity analyses for scientific models. Morris, (1991) developed a on-at-atime elementary effects method. This has been extended e.g. towards variance-based sensitivity analyses, a global sensitivity analysis (Sobol', 2001). Authors like Campolongo et al., (2007), Campolongo et al., (2011) further extended the approach and its application and partly also developed tools for supporting the sensitivity analyses (e.g. Pianosi et al., (2015), (Herman et al., 2020). Among others it was the reason of computational time constraints which made us take the decision for the Morris method and applying the SAFE-toolbox (Pianosi et al., 2015).

The application of global sensitivity models in building stock modelling is not common practice. One of the rather rare examples is (Branger et al., 2015).

Within the IEA EBC Annex 70 on "Building Energy Epidemiology" a group of building stock modelling teams applied different approaches of global sensitivity analyses and exchanged results and experiences. This work also was carried out in the frame of this exchange.

In this paper we apply the building stock model Invert/EE-Lab (www.invert.at), using the Elementary Effects Method as a global sensitivity analysis, a method appropriate for the

degree of complexity of the model. We provide exemplary results for selected parameters for selected countries. The method and discussion of parameter selection is based on the IEA EBC Annex 70 - Building Energy Epidemiology project. Invert/EE-Lab techno-sociois а economic bottom-up building stock model which has been applied in more than 40 projects in EU-27+ countries (Müller, 2015), (Kranzl et al., 2018). The model builds on input data including disaggregated building stock database on country level, supply technologies, regional climate, energy prices and energy carrier

potentials as well as behavioural aspects and

investment

decision

criteria. Within this article we focus on the analysis of the influence of relevant indicators such as interest rates, costs, energy prices, selected technical parameters and behavioural aspects on the final energy demand, respectively related energy carrier shares or installed capacities.

Methods

As explained above, in this paper we apply the elementary effects method in line with (Morris, 1991) as a means of sensitivity analysis. The Elementary Effects method can be seen as a randomized "One-At-a-Time" design. Thus, for every model run one input parameter is modified by a random sampling method. Elementary effects for each input are computed from different points in the input space, leading to mean and standard deviation that can be taken as a measure of importance of a specific input variable and its interactions with other inputs. It's important to note that both results, i.e. the mean and standard deviation are required to consider for the conclusion. This method is applied to the building stock model Invert/EE-Lab, which is described below and in Figure 1.

For the application of the EE-method to Invert/EE-Lab we use the SAFE toolbox (Pianosi et al, 2015) to generate the input samples used for the model iterations as well as for analysis and visualization. The great variety of input



Figure 1 Overview structure of the simulation tool Invert/EE-Lab

Table 1. Input parameters considered in the global sensitivity analysis

		Method (fixed val., add, mult)		Distr.	General variation		Specific values for estimated bandwidth	
group	parameter	fBWf	spV		xmin fBWf	xmax fBWf	xmin spV	xmax spV
Decision process	interest_rate (hs change / ren)	m	f	unif	0.7	1.3	1%	12%
	economic weight (hs change / ren)	m	f	unif	0.7	1.3	0.6	0.9
	subsidy awareness	m	f	unif	0.7	1.3	0.5	2
	lambda	m	f	unif	0.7	1.3	2	6
Costs	heating systems costs	m	m	unif	0.7	1.3	0.75	1.25
	renovation costs	m	m	unif	0.7	1.3	0.75	1.25
Energy prices	energy_prices	m	m	unif	0.7	1.3	0.85	1.15
Technical	lifetime (building & comp.)	m	m	unif	0.7	1.3	0.8	1.2
	COP heatpumps	m	m	unif	0.7	1.3	0.75	1.3
User behaviour	service factors	m	f	unif	0.7	1.3	0.5	1
dummy	dummy	m	f	unif	0.7	1.3	0.8	1.2

parameters was broken down to 11 parameters, comprising of *interest rates*, *different investment decision parameters, heating system and renovation costs, energy prices, lifetime of buildings and heat pump coefficient of performance, as well as factors representing user behaviour* (see explanation and the rationale for their selection below).

Invert/EE-Lab

Invert/EE-Lab is a dynamic techno-socio-economic bottom-up building stock model that evaluates the effects of different framework conditions (in particular different settings of economic and regulatory incentives) on the total energy demand, energy carrier mix, CO2 reductions and costs for space heating, cooling and hot water preparations in buildings. The model describes the building stock, heating, cooling and hot water systems on highly disaggregated level, calculates related energy needs and final energy demand, determines reinvestment cycles and new investment of building components and technologies and simulates the decisions of various agents (i.e. owner types) in case that an investment decision is due for a specific building segment. The core of the tool is a myopical, multinominal logit approach, which optimizes objectives of "agents" under imperfect information conditions and by that represents the decisions maker concerning building related decisions. Myopical in this context means that the model does not assume a perfect foresight rationale of the decision makers. Rather, they assume that the framework conditions affecting the decision will remain constant in the future, although in reality – and in the scenario model run - this may not be the case. The model applies a nested logit approach in order to calculate market shares of heating systems and energy efficiency measures depending on building and investor type. Invert/EE-Lab covers residential and non-residential buildings. The model has been applied in more than 40 projects for various countries and for EU-27 (+UK and selected neighboring countries), among others also for the European Commission.

More information is available at www.invert.at or e.g. in (Müller, 2015)) and (Kranzl et al., 2018).

Standard outputs from the Invert/EE-Lab on an annual basis are:

• Installation of heating and hot water systems by energy carrier and technology (number of buildings, number of dwellings supplied)

• Refurbishment measures by level of refurbishment (number of buildings, number of dwellings)

- Total delivered energy by energy carriers and building categories (GWh)
- Total energy need by building categories (GWh)
- Policy programme costs, e.g. support volume for investment subsidies (M€)
- Total investment (M€)

Uncertainty analysis: OAT and the SAFE Toolbox

As a method for evaluation and comparison of the Sensitivity/Uncertainty analysis we carried out several variable variation variants using the Elementary Effects method by Morris (Campolongo et al., 2007), calculating mean and standard deviation of elementary effects.

The analysis was implemented in the SAFE Toolbox (MATLAB), complemented by data management scripts written in Python (https://www.safetoolbox.info/info-and-documentation/).

The implementation process of the sensitivity analysis was carried out in the following steps:

- 1) Generate input data structure (Python)
- 2) Generate input data sample (SAFE)
- 3) Adapt input data (Python)
- 4) Run model Invert/EE-Lab (Multiple model runs per sample and averaging of output values to reduce impact of model stochastics)
- 5) Get mean results of output parameters (Python)
- 6) Compete elementary effects and visualize results (SAFE)

For the selected input parameters (see), after generating the input data structure based on Invert/EE-Lab data, the SAFE toolbox has been used to generate the input data sample, for which latin hypercube has been selected as sampling strategy. This modified data sample is used to modify Invert/EE-Lab input data, which is run multiple times for the created input data samples. For the resulting output parameters indicators are created and visualized. shows the number (M) of selected 11 input parameters (including one dummy for testing purposes).

As can be seen from , the number of possible input parameters which could be selected for the sensitivity analysis is high. It ranges from the description of the building stock, to techno-economic data of heating systems and renovation measures, energy prices, various restrictions. policy instruments, user behaviour, preferences or parameters affecting the decision process. First, we neglected parameters which mainly have an impact on the current state of the building stock. Although we are aware that there are significant uncertainties in the description of the current stock, in principle it is well calibrated e.g. against national energy balances. Second, we decided to analyse the impact on heat pump related outputs like installed power of heat pumps. Thus, we also selected some parameters with obvious (or potential) effect on the model's heat pump related outputs like COP of heat pumps, energy prices, including electricity prices and heating system costs. Renovation activities (driven strongly by renovation costs) in the model have an impact on the COP of heat pumps and thus on the economic viability and diffusion of heat pumps. This is why they also have been chosen. Third, from previous analysis (e.g. Müller, 2015) the authors know that the model shows some sensitivity on user behaviour and certain parameters affecting the decision process, although the scale of this impact was not yet known. Thus, we selected these parameters as well.

We decided to evaluate two variation sets: (1) a general variation: sample within a -/+ 30% margin and (2) a variation within a range based on expert guess for each of the parameters. In the second case we acknowledged the fact that the possible variation of some parameters probably is not symmetric but may be larger on the right hand side of the mean value.

The number of elementary effects was varied as r = 20, 30, 40, 100. The final number of model evaluations results in $r^{*}(M+1)$. Due to the fact that Invert/EE-Lab shows some stochastic behaviour (Müller, 2015), we carried out 3-5 model runs per sample and calculated the mean output value of these model runs (2nd box from the right in). Overall, for the case of France, this led to a number of 20400 model runs.

Due to the restricted space availability in the paper, as output-variables we selected the following: installed power of heat pumps in the year 2030 and final energy consumption of gas for space heating and hot water in 2030.

Selected scenarios and countries

In order to show the applicability of the uncertainty method indicated above, we selected existing scenarios for the countries Sweden and Spain. The scenarios have been developed in the project SET-Nav and the scenario logic and background scenario data of this so-called "reference scenario" is described in more detail in (Hartner et al., 2018) and Crespo del Granado et al., (2020). The selected scenario shows the impact of current policies in place (status 2015), and thus are in contrast to strong (or even complete) decarbonisation scenarios. However, still some significant changes in the technology mix and energy demand occur by 2050. Due to the fact that the main intention of this paper is to demonstrate the applicability of global sensitivity analysis in this type of model we decided to show results only for the year 2030, in order to reduce computation time.

Results

The following figures show the results for the selected output variables for the case of Sweden (Figure 2 to Figure 5) and Spain (Figure 6 to Figure 9).

For each output variable (and each for the case of fixed bandwidth factor and specific values for the input parameter variation sampling) we present a graph of the standard deviation of the elementary effects over the mean of the elementary effects. Thus, in each of the figures, the impact of the variation of the selected input parameters (Table 1) on a certain output parameter (described above) is indicated. The x-axis shows the mean of the elementary effect (EE), i.e. how strongly the output parameters are varying with varying input parameters. The v-axis shows the standard deviation of EEs. i.e. how strongly the EE deviate in the whole set of model runs while also varying the other input parameters. Thus, the y-axis is also an indicator for how strongly the impact of an input parameter depends on other input parameters. Since this is done in each figure for each of the 11 selected input parameters, we can identify those parameters with the highest impact on standard deviation and mean of the elementary effect.

Figure 2 shows the first exemplary result graph. The dummy variable (red circle), which was introduced for testing purposes should show no elementary effect. The reason for the slightly positive mean and standard deviation of the elementary effect is the stochastic behaviour of the model. Although we derived the mean of several model runs for each set of input parameters, a slightly positive effect is still visible. This should also be considered for the other result graphs.¹

¹ In the discussion section we examine to which extent the stochasticity of the model distorts the conclusions and what would need to be done as further work in order to

separate the impact of the model stochasticity from the sensitivity analysis due to the variation of input parameters.

Sensitivity analysis results for the case of Sweden



Figure 2 Sensitivity analysis results, installed power of heat pumps in 2030, fixed bandwidth factor, Sweden²



Figure 3 Sensitivity analysis results, final energy consumption of gas for space heating and hot water demand in 2030, fixed bandwidth factor, Sweden

Output parameter "installed power of heat pumps"

For the output parameter "installed power of heat pumps" and in case of the "specific values of parameter variation", the following two variables prove to be the most sensitive in both countries (i.e. Figure 2 and Figure 4 for SWE and Figure 6 and Figure 8 for ESP, for fixed bandwith and specific values, respectively): (1) "**awa s**" is a factor that determines the level of **awareness of support instruments**, i.e. to which extent decision makers are aware that support instruments exist and may be an attractive incentive. Obviously, the existence of a support scheme for heat pumps and its awareness among building

owners is a significant factor. In both countries the scenario setting assumes that such support schemes are in place. However, the factor "awa sub" determines how well they are communicated and how effective the scheme works. (2) The lambda value represents the coefficient in the logit approach of the model that describes the sensitivity of building owners' decision behaviour with regard to the reaction to differences between heating system options. Thus, if two heating systems have very similar but slightly different costs (or other properties relevant for the utility function), the lambda factor determines how strongly different actors respond to this difference when choosing a heating system. The high sensitivity of the lambda factor thus shows that some heating systems have very similar costs (or other properties relevant to the utility function) and that it is important to interpret the model results accordingly.

However, the comparison with the result graphs "fixed bandwidth factor" for the output parameter "installed heat pumps" reveals that this is largely driven by the assumptions for the parameter ranges chosen according to Table 1. For both countries, other input parameters show also comparable sensitivity, for Spain even the first ranked parameter changed: only for "specific values" the lambda values is the first ranked parameter.



Figure 4 Sensitivity analysis results, installed power of heat pumps in 2030, specific values, Sweden

technologies; hs cost - heating system cost; ren cost - renovation cost; e price - energy price; Lt - lifetime of heating systems; cop hp - annual performance factor of heat pumps; serv f - service factor that determines to which extent buildings are heated; dummy - dummy parameter for testing purposes.

² The following abbreviations apply for the following figures: IR - interest rate; eco w - weight of the economic rational calculus in decision making regarding building renovation and heating system choice; awa sub - subsidy awareness - factor determining the degree of awareness of subsidy instruments; lambda - coefficient in the logit approach for determining the market share of



Figure 5 Sensitivity analysis results, final energy consumption of gas for space heating and hot water demand in 2030, specific values, Sweden

Output parameter "final energy consumption of gas"

As far as the final energy consumption of **natural gas for** space heating and domestic hot water is concerned, in addition to the previously mentioned input parameters, the following also turn out to be sensitive: In the case of Sweden, the variation of the energy price as well as the coefficient of performance (COP) of heat pumps shows a relatively high effect. On the one hand, this is due to the fact that the energy price level is already relatively high due to taxation (at least higher than in Spain). On the other hand, the importance of the model sensitivity to the annual performance factor of heat pumps shows that heat pumps are a relevant competitor to natural gas in the model results, although the annual performance factor plays a not insignificant role in how attractive heat pumps are compared to gas boilers. In the case of Spain, it is the lifetime of heating systems that shows a significantly higher influence on the results than the variation of other input parameters. This is because there is a higher stock of gas heating systems compared to Sweden. Depending on when these are due for replacement due to reaching their service life, there is a more or less rapid replacement of gas boilers with renewable heating systems.

For the case of "fixed bandwidth", the service factor turns out to be the most sensitive input parameter. The service factor in the model Invert/EE-Lab determines part of the effect that often, only a certain share of the space heating energy demand is actually covered by the heating system. I.e. the higher the service factor, the higher is the comfort level and the higher the effective indoor temperature during the heating season. The factor depicts the part of this effect which in reality is triggered by the type of heating system. E.g. automatized, central heating systems typically trigger a higher service factor than manually fed wood log single stoves. On the one hand, the factor is used for considering this difference between heating system observed. On the other hand, this is one of the factors which the authors apply for calibrating the model to national energy balances. Still, despite the fact that there is some empirical evidence available, e.g. (Haas et al., 1998), the choice of this parameter in particular for different household types and regions across Europe is subject to uncertainty. At least for the case of fixed bandwidth parameter variation, the analysis shows a relevant impact of this parameter on the final energy consumption. This is a plausible result, since the overall level of energy consumption is affected by this factor.

Sensitivity analysis results for the case of Spain



Figure 6 Sensitivity analysis results, installed power of heat pumps in 2030, fixed bandwidth factor, Spain



Figure 7 Sensitivity analysis results, final energy consumption of gas for space heating and hot water demand in 2030, fixed bandwidth factor, Spain



Figure 8 Sensitivity analysis results, installed power of heat pumps in 2030, specific values, Spain



Figure 9 Sensitivity analysis results, final energy consumption of gas for space heating and hot water demand in 2030, specific values, Spain

Discussion and conclusions

The analysis was carried out with respect to the output variables *installed capacity of heatpumps* and *final energy consumption of natural gas heating systems*. The results for installed *heatpump capacity* show different levels of importance and interconnections for the evaluated input variables. Whereas the parameter *economic weight* can be considered to be of significant influence and great interconnection with other variables, others like *interest rates*, *costs* and *behavioural factors* can be considered as less influential on the observed output variable. It has to be mentioned tough, that the results are highly dependent on the selected value ranges for the input parameters, as well as on country specific datasets and pre-sets. This becomes clear when comparing the results for the "specific values" parameter variation and the "fixed bandwidth" variation. In our analysis, it was not possible to determine the choice of the range of parameter variation by means of empirical, historical values. For some of the input parameters, like energy prices or COPs this would be feasible. For others, it would mean substantially higher effort. Thus, overall we need to keep this for further research. As long as such an empirical substantiation of parameter variation ranges is not possible, the consideration of both fixed bandwidth variation and specific values – defined on the basis of expert estimations – both are important for determining the parameters with the highest sensitivity on a certain output parameter.

The high relevance of the logit-coefficient lambda (Müller, 2015) on some output parameters indicates that the scenario results regarding heating system (and related energy carrier) mixes for technologies with relatively similar costs should be considered with caution³. And it shows the need for further research of understanding the decision making behaviour of different types of building owners and how this can be integrated in building stock models.

As described above, we can observe a slightly positive value of the dummy variable due the stochastic behaviour of the model. In order to clearly identify and quantify the impact of the stochastic model behaviour on the sensitivity results, it would be required to identify the range of output parameters resulting from this effect. However, since this would need to be done for the significant amount of scenario results, we need to keep this task for further research. Müller (2015) described the stochastic behaviour of the model in more detail. In particular, he showed that final energy demand results are close to a normal distribution. For the settings chosen in our analysis for the stochastic behaviour we estimate that the role of the stochastic behaviour of the model is much less relevant than the variation of (some of) the selected input parameters. This is underlined by the fact that the dummy value shows comparably low EE mean and EE standard deviation results.

Overall, the sensitivity analysis carried out through the elementary effects method provides valuable insights for the influence of various input parameters. The insights gained can be used for improved scenario development as well as deeper result interpretation trough better model understanding.

The analyses presented in this paper were done only for the model Invert/EE-Lab. We assume that similar conclusions can be derived for similar type of technosocio-economic bottom-up building stock models. However, the exact behaviour of the models and

³ On the other hand, the high variation that the authors suggested to apply for the case of "specific values" for the lambda value (table 1), can also be considered as prejudgement, since the authors were already aware of the

relevance and uncertainty of this parameter. For the "fixed bandwidth" variation, the impact of the lambda value at least for some output parameters is much smaller.

comparison of the sensitivity of different models remains an open question for further research activities.

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