Coupling Flexible Structure and Viscous, Compressible Fluid: A Case for Model Order Reduction?

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Considering viscous effects in acoustics is important in cases where the size of the viscous boundary layer is not negligible. This is often the case on small scales, e.g. in MEMS devices, or small features. We employ the finite element method to solve the governing equations: the linearized mass and momentum conservation equations for a viscous, compressible fluid, coupled to the balance of momentum for a linear elastic solid. Since the computational effort is high, we attempt a model order reduction procedure using the standard reduced basis method.

We consider a 2D example of an elastic, fluid-filled container with two chambers connected by a narrow neck. The container is placed on top of a flexible pillar, which is excited at its base. Dimensions and material parameters are selected to ensure strong fluid-structure interaction.

The formulation is implemented in the open source FEM software openCFS.

Find $p$, $v$, and $u$ such that

$$\int_{\Omega} \sum_{i} \left( \int_{\Omega} \frac{1}{2} \rho \frac{\partial v_i}{\partial t} \cdot \frac{\partial v_i}{\partial t} - p \cdot \nabla v_i \cdot v_i - \frac{1}{2} \frac{\partial p}{\partial t} \right) \, \text{d}V + \int_{\Gamma} \sum_{i} \left( \int_{\Gamma} \frac{1}{2} \mu \frac{\partial v_i}{\partial n} \cdot \frac{\partial v_i}{\partial n} + \frac{1}{2} \frac{\partial u_i}{\partial n} \cdot \frac{\partial u_i}{\partial n} \right) \, \text{d}S = 0$$

for all test functions $p_i$, $v_i$, and $u_i$.

System Behaviour

Direct solution of the full system ...

Model Order Reduction

using the standard reduced basis method

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{uu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{vw} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{vv} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{uu} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{vw} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{vv} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_{ww} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{yy} & 0 \end{bmatrix} \begin{bmatrix} p \\ v \\ w \\ \beta \\ u \\ \nu \\ \alpha \\ \gamma \\ \delta \\ \epsilon \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

for different dynamic viscosities $\mu$, $\mu 4$, and $\mu 16$.

The solution of the full system is computationally expensive. Distinct peaks are clearly visible in the transfer functions, and the viscosity determines the amount of damping. Despite the badly conditioned system matrices the results seem accurate and physically plausible.

Model order reduction using the standard modal basis approach does not work for the coupled system. Possibly a suitable reduction base can be found from the modes of the un-coupled fluid and solid systems, respectively. If you have suggestions, do not hesitate to get in touch...