Broad excitations in a 2+1D overoccupied gluon plasma
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FunQCD 2021, March 31, 2021

Talk mainly based on: arXiv:2101.02715
PRD 100, 094022 (2019), [arXiv:1907.05892]
Table of Contents

1 Introduction

2 Models, setup and a new attractor (1907.05892)

3 Excitation spectra of gluon plasmas (1804.01966, 2101.02715)

4 Conclusion
The goal is to study microscopically properties of QCD nonperturbatively

Microscopic properties of QCD nonperturbatively

⇒ Spectral functions $\rho(\omega, p)$ of gluons / quarks encode full excitation spectrum!

- Our approach: far from equilibrium
  - Highly occupied gluon plasma ($A \sim 1/g$), weak coupling ($g^2 \ll 1$)
  - Then nonperturbative and perturbative methods available!

- Classical-statistical lattice simulations vs. HTL, kinetic theory
  
  class. covariant equ.: $D_\mu F^{\mu\nu} = 0$, \quad HTL: $\Pi^{HTL}_{\mu\nu}$

- Nonequilibrium application: heavy-ion collisions
- General qualitative features? (e.g., relevance for thermal equ.?)
Motivation: heavy-ion collisions

Application

Microscopic properties of non-equilibrium QCD

✓ Initial stages in heavy-ion collisions suitable playground
✓ Quark-gluon plasma initially: eff. 2+1D and 3+1D mainly gluonic
  - Initially color fields approx. boost invariant (Glasma) ⇒ eff. 2+1D
  - Later: Bottom-up scenario with kinetic theory in anisotropic 3+1D
★ Excitation spectra: Quasiparticles (QP)? When is kinetic theory valid?
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3 Excitation spectra of gluon plasmas (1804.01966, 2101.02715)

4 Conclusion
**Considered models and initial conditions**

- **SU($N_c$) Yang-Mills theory (simulations: $N_c = 2$)**
  \[
  S_{YM}[A] = -\frac{1}{4} \int d^{d+1}x \, F^{\mu \nu}_{a} F^{a}_{\mu \nu}
  \]
  (in gauge-covariant formulation, with links $U_j \approx \exp(ig a_s A_j)$)

- **Models of initial stages (here non-expanding geometry)**
  1. **2+1D**
  2. **Glasma-like 2+1D**: add adjoint scalar $\phi$ to model 1
  3. **3+1D**: isotropic 3+1D

- **Initial conditions (with $E = \partial_0 A$):** highly occupied
  \[
  f(t = 0, p) = \frac{Q}{g^2} n_0 e^{-\frac{p^2}{2Q^2}} \quad \text{with} \quad f(t, p) \propto \frac{\langle |E_T(t, p)|^2 \rangle}{p}
  \]

- **Solve classical equations, average over initial ensembles**
  \[\Rightarrow\] classical-statistical lattice simulations
Classical-statistical lattice simulations

1. Set initial conditions

$$\langle E_T^*(t_0, \vec{p}) E_T(t_0, \vec{q}) \rangle \propto p f(t_0, p) \delta_{jk} (2\pi)^3 \delta(\vec{p} - \vec{q})$$

with $E^j_T p_j = 0$, initially $\langle E_L^*(t_0, \vec{p}) E_L(t_0, \vec{q}) \rangle = 0$, same for $A$


3. Solve classical field equations on the lattice

$$U_j(t + dt/2, \vec{x}) = e^{idt a_s g E^j_a(t, \vec{x})} U_j(t - dt/2, \vec{x})$$

$$gE^i_a(t + dt, \vec{x}) = gE^i_a(t, \vec{x}) - \frac{dt}{a^3} \sum_{j \neq i} \left[ U_{ij} \left( t - \frac{dt}{2}, \vec{x} \right) + U_{i(-j)} \left( t - \frac{dt}{2}, \vec{x} \right) \right]_{ah}$$

4. Evolve each initial configuration $\{ U(t_0, \vec{x}), E(t_0, \vec{x}) \}$ until $t > t_0$

5. Compute observable $O[U, E]$ that depends on the fields

$$O(t) = \frac{1}{\# k} \sum_k O[U(t), E(t)]$$
Both 2+1D theories approach a classical self-similar attractor

\[ f(t, p) = (Qt)^\alpha f_s \left((Qt)^\beta p\right) \]

Universal scaling exponents insensitive to details of initial conditions

\[ \beta = -\frac{1}{5}, \quad \alpha = 3\beta; \quad \text{(parametric kinetic explanation in Backup)} \]

Hard scale \( \Lambda(t) = \langle p \rangle \sim Q(Qt)^{-\beta} \)

Classical attractor in 3+1D well known, \( \beta = -\frac{1}{7}, \quad \alpha = -\frac{4}{7} \)
Equal-time correlator $\langle \{ \hat{E}(t), \hat{E}(t) \} \rangle \propto f(t, p)$ is distribution

$\Rightarrow$ But what are the relevant excitations?

Knowledge of spectral function needed ($\dot{\rho} = \partial_t \rho$, $E = \partial_t A$)

$$\dot{\rho}(x, x') = \frac{i}{N_c^2 - 1} \left\langle \left[ \hat{E}(x), \hat{A}(x') \right] \right\rangle$$

Statistical correlator $\langle EE \rangle \equiv \ddot{F}$ in general independent of $\dot{\rho}$

$$\langle EE \rangle(x, x') = \frac{1}{2(N_c^2 - 1)} \left\langle \{ \hat{E}(x), \hat{E}(x') \} \right\rangle$$

Fourier transf. in $t - t'$ and $\vec{x} - \vec{x'}$ to frequency $\omega$ and momentum $\vec{p}$

Approximation: normally at fixed $\bar{t} = \frac{1}{2}(t + t')$, we hold $t \approx \bar{t}$

In classical-statistical simulations

$$\langle EE \rangle(t, t', p) = \frac{1}{N_c^2 - 1} \left\langle E(t, \vec{p})E^*(t', \vec{p}) \right\rangle$$

Gauge: temporal $A_0 = 0$ + Coulomb-type $\partial_j A_j \big|_t = 0$
Perturbative computation: HTL results

- Hard loop (HTL) framework applicable for $m_D/\Lambda \ll 1$; in thermal equ. for $g \sim m_D/T \ll 1$, Braaten, Pisarski (1990); Blaizot, Iancu (2002); ...

- In 3+1D $m_D^2 = 4N_c \int \frac{d^3p}{(2\pi)^3} \frac{g^2f(t,p)}{p} \sim g^2f\Lambda^2 \Rightarrow$ HTL applicable

- In 2+1D soft-soft interactions important

$$m_D^2 \approx d_{pol}N_c \int \frac{d^2p}{(2\pi)^2} \frac{g^2f(t,p)}{\sqrt{m^2 + p^2}} \sim g^2f\Lambda \ln \left(\frac{\Lambda}{m_D}\right)$$

$\Rightarrow$ HTL breaks down already at soft scale $p \sim m_D$

- Comparison to HTL still useful to extract nonperturbative features

- Quasiparticles in $\rho^{HTL}(\omega, p)$ as $\sim \delta(\omega - \omega^{HTL}_{\alpha}(p))$

- All expressions depend only on $m_D$, computed consistently in HTL
Nonperturbative computation of spectral function $\rho$

Classical-statistical $\text{SU}(N_c)$ simulations + linear response theory

- Perturb $A(t, \vec{x}) \mapsto A(t, \vec{x}) + \delta A(t, \vec{x})$
- Class. EOM for $A$: $D_\mu F^{\mu\nu}[A] = 0$
- Linearized EOM for $\delta A(t, \vec{x})$
  (also in gauge-cov. formulation)
  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $G_R(t, t', p) \propto \langle \delta A(t', \vec{p}) \delta E^*(t, \vec{p}) \rangle$
- $\theta(t - t') \left[ \rho(t, t', p) \right] = G_R(t, t', p)$

Similar methods for scalars:
Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020)
Isotropic 3+1D gluon plasmas


- Narrow Lorentzian quasiparticle peaks for all momenta, even $p \lesssim m_D$
- Generalized fluctuation dissipation relation (FDR) for $\alpha = T, L$

$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t=0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t=0, p)}$$

- Width $\gamma_\alpha(p) \ll \omega_\alpha(p)$, decreases $\gamma_\alpha(t) \sim (Qt)^{-2/7} m_D(t)$
Spectral function in isotr. 3+1D


- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

$\gamma_{T/L}(p)$ beyond HTL at LO

First determination of $p$ dep.

‘isotropic’ $\gamma_T(p) \approx \gamma_L(p)$

HTL prediction $\gamma(p = 0)$

Braaten, Pisarski, PRD 42, 2156 (1990)
Generalized FDR observed

Broad peaks $\gamma_\alpha \sim \omega_{pl} \equiv \omega_T(p=0) \propto m_D$  

$\Rightarrow$ no quasiparticles for $p \lesssim m_D$!
Now: correlations in $2+1$D theories (2101.02715)

- Generalized FDR observed
- Broad peaks $\gamma_\alpha \sim \omega_{pl} \equiv \omega_T(p=0) \propto m_D$
  
  \[ \Rightarrow \text{no quasiparticles for } p \lesssim m_D! \]
- Non-Lorentzian shape of the peaks (Backup)
- HTL curves (green) agree poorly (except for $\omega \ll p$ for long.)
- At low $p$: for $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite, $\rho_T \sim 1/\omega$
Time dependence

- \( \omega_{pl} \dot{\rho}(t, \omega/\omega_{pl}, p/\omega_{pl}) \) is time independent (for all \( p \), Backup)
- This implies \( \gamma_\alpha(t, p) \sim \omega_{pl}(t) \sim m_D \sim Q(Q_t)^{-1/5} \)
- Estimates as for 3+1D lead to \( Q(Q_t)^{-2/5} \) ⇒ different mechanism
Time dependence

- \( \omega_{\text{pl}} \dot{\rho}(t, \omega/\omega_{\text{pl}}, p/\omega_{\text{pl}}) \) is time independent (for all \( p \), Backup)
- This implies \( \gamma_\alpha(t, p) \sim \omega_{\text{pl}}(t) \sim m_D \sim Q(Qt)^{-1/5} \)
- Estimates as for 3+1D lead to \( Q(Qt)^{-2/5} \Rightarrow \) different mechanism
- Also in classical thermal equilibrium \( \gamma \sim \omega_{\text{pl}} \) (Backup)
- No quasiparticles at low \( p \) seems to be quite general in 2+1D
Fermion spectral function

$$\rho(t, \omega, \vec{p}) \approx \gamma^0 \rho_0^V(t, \omega, \vec{p}) + \gamma^i \frac{p^i}{p} \rho_V(t, \omega, \vec{p})$$

$$\rho_+ = \rho_0^V + \rho_V$$
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# Conclusion
Conclusion

- Equal-time correlators often insufficient to understand microscopics
  ⇒ unequal-time correlations required
- In 2+1D, new classical attractor observed and $\rho$ extracted
- Much broader peaks in 2+1D than in 3+1D
  ⇒ excitations too short-lived to form quasiparticles for $p \lesssim m_D$
  (an effective kinetic description may be possible for $p \gg m_D$ but requires nonperturbatively determined collision kernel)

Outlook

- **Gauge-invariant checks** of nonperturbative properties:
  Heavy-quark diffusion KB, A. Kurkela, T. Lappi, J. Peuron, JHEP 09, 077 (2020), transport, ...
- $\rho$ in Bjorken expanding systems
- How does an eff. **kinetic theory in 2+1D** gauge systems look like?
Thank you for your attention!

3+1D

2+1D
Backup slides
Universal (classical) attractors

- Rich nonequilibrium dynamics in gauge and scalar systems
- Share similar universal features:

**Nonthermal fixed point (NTFP)**

- Large initial occupancy \( \Rightarrow \) may approach attractor
- System ‘forgets’ initial conditions
- Self-similar dynamics

\[
f(t, p) = t^\alpha f_s(t^\beta p)
\]

- Universal \( \alpha, \beta, f_s(p) \)

**NTFP:** Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)

**Universality:** Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)

**Experimental observations:** Prüfer et al., Nature 563, 217 (2018); Erne et al., Nature 563, 225 (2018)
Self-similarity of 2+1D theory (PRD 100, 094022 (2019))

- **Self-similar evolution**

\[ f(t, p) = (Qt)^\alpha f_s \left((Qt)^\beta p\right) \]

- **Universal scaling exponents**

\[ \beta = -\frac{1}{5}, \quad \alpha = 3\beta \quad (\text{energy conserv.}) \]
Perturbative explanation of scaling exponents (PRD 100, 094022 (2019))

- Soft scale (Debye mass) from HTL

\[ m_D^2 \approx d_{\text{pol}} N_c \int \frac{d^2 p}{(2\pi)^2} \frac{g^2 f(t, p)}{p} \sim g^2 f \Lambda \ln(\Lambda/m_D) \]

(Log from soft-soft interactions ⇒ breakdown of HTL at \( m_D \) in 2+1D)

- Scaling exponents from kinetic arguments:

  Elastic scattering rate:

  \[ \frac{d\Gamma}{dq_\perp} \sim \frac{g^4}{(q_\perp^2 + m_D^2)^2} \int d^2 p f(1 + f) \]

  Momentum diffusion:

  \[ \hat{q} \sim \int dq_\perp \frac{d\Gamma}{dq_\perp} q_\perp^2 \sim \frac{\Lambda^2 (g^2 f)^2}{m_D} \]

  From broadening \( \Lambda^2 \sim \hat{q} t \) follows \( \Lambda \sim Q(Qt)^{1/5} \)

- In 2+1D \( q_\perp \sim m_D \) crucial! If \( q_\perp \sim \Lambda \), then \( \Lambda \sim Q(Qt)^{1/7} \) instead!

- Kinetic estimates work ⇒ eff. kinetic descr. may exist for \( p \gg m_D \)
**Left:** Different ways of computing the Fourier transform are consistent

**Right:** Peak has non-Lorentzian shape (not Breit-Wigner)
Dispersion relations, damping rates in Glasma-like 2+1D (arXiv:2101.02715)

- **Left:** Dispersions $\omega_\alpha(t, p)/\omega_{pl}(t)$
- **Right:** Peak width $\gamma_\alpha(t, p)/\omega_{pl}(t)$
- As functions of $p/\omega_{pl}(t)$ time independent $\Rightarrow \gamma(t, p) \sim \omega_{pl}(t)$
- Scalar excitation narrow for $p \lesssim m_D$, but same $t$ dependence
Qualitatively similar behavior as far from equilibrium

- Broad gluonic excitations with $\gamma(p) \sim \omega_{pl}$
- HTL provides poor description
- For $\omega \to 0$, $\dot{\rho}_T = \omega \rho_T$ finite at low $p$

Interpretation:

These qualitative features seem generic in 2+1D gauge theories
Summary of $\langle EE \rangle_\alpha (t=\text{const}, \omega, p)$

Black dashed: $\omega^\text{HTL}_\alpha (p)$,  
gray: $\sqrt{\omega^2_{\text{pl}} + p^2}$, with $\omega_{\text{pl}} \propto m_D$
Heavy quark diffusion: excess of IR gluons in isotropic 3+1D

The following is based on

- What are the consequences of the nonperturbative properties of correlation functions?
- E.g., consider correlations in 3+1D at self-similar attractor

Excess of infrared gluons

\[ \langle EE \rangle_{T/L}(t, t, p) \text{ shown} \]

- Dashed: HTL expectations
- Excess of gluons w.r.t. HTL for \( p \lesssim m_D \sim \omega_{pl} \)
Heavy quark diffusion: far from equilibrium in 3+1D

- Correlations like $\langle EE \rangle_T(t, t', p)$ are not gauge-invariant
- We used $A_0 = 0$ and $\vec{\nabla} \vec{A} = 0|_t$ gauges
- Are effects visible in gauge-invariant observables?

**Example**

Heavy quark in QGP

- Quark experiences ‘kicks’ from the medium
  \[ \dot{p}_i(t) = F_i(t) \]
- Gauge-inv. force-force correlator leads to momentum broadening
  \[
  \langle \dot{p}_i(t) \dot{p}_i(t') \rangle = g^2 \frac{\text{Tr} \langle E_i(t, \vec{x}) U_0(t, t', \vec{x}) E_i(t', \vec{x}) U_0(t', t, \vec{x}) \rangle}{\text{Tr} \mathbb{1}} \\
  = \frac{g^2}{2N_c} \langle E_i^a(t, \vec{x}) E_i^a(t', \vec{x}) \rangle \equiv \frac{g^2}{2N_c} \langle EE \rangle(t, t')
  \]
Models to understand evolution of $\kappa(t, \Delta t)$

SR: ‘Spectral reconstruction’

$$3\kappa(t, \Delta t) = \frac{d}{d\Delta t} \langle p^2(t, \Delta t) \rangle$$

$$\approx g^2 \int \frac{d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega}$$

$$\times \left[ 2\langle EE \rangle_T(t, t, p) \frac{\dot{\rho}_T(t, \omega, p)}{\dot{\rho}_T(t, t, p)} + \langle EE \rangle_L(t, t, p) \frac{\dot{\rho}_L(t, \omega, p)}{\dot{\rho}_L(t, t, p)} \right]$$

- use extracted equal-time and spectral functions in computation
- use $\langle EE \rangle_{T/L}(t, t, p)$ with IR excess (‘data’) & without (‘thermal HTL’)
- in $\dot{\rho}_{T/L}(t, \omega, p)$ Landau ($\omega < p$) and q.p. terms can be distinguished
Heavy-quark diffusion: IR gluon excess observable

\[ \kappa(t, \Delta t) \approx \frac{g^2}{3N_c} \int \frac{d^3p}{(2\pi)^3} \int_\infty^{-\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \times \left[ 2\langle EE \rangle_T(t, t, p) \frac{\dot{\rho}_T(t, \omega, p)}{\dot{\rho}_T(t, t, p)} + \langle EE \rangle_L(t, t, p) \frac{\dot{\rho}_L(t, \omega, p)}{\dot{\rho}_L(t, t, p)} \right] \]

Total \( \kappa(t, \Delta t) \equiv \sum_i \kappa_i(t, \Delta t) \)

Components \( \kappa_i(t, \Delta t) \)

- Nonperturbative effects of \( \langle EE \rangle_\alpha(t, t, p) \) and \( \dot{\rho}_\alpha(t, \omega, p) \) visible!
- Oscillations with \( \omega_{p1} \) due to QP excitations, sign of IR excess
- Heavy quarks, quarkonia, jets may reveal IR dynamics of QGP