Varieties of logic as algebra and some of their empirical realisations WS From Organizations to Goal-Directedness: Systemic and Interdisciplinary Modeling. http://tph.tuwien.ac.at/~svozil/publ/ 2022-Granada-pres.pdf

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Empirical nonclassical logic

1. Birkhoff & von Neumann "The Logic of Quantum Mechanics" (1936) DOI 10.2307/1968621 "One of the aspects of quantum theory which has attracted the most general attention, is the novelty of the logical notions which it presupposes. It asserts that even a complete mathematical description of a physi- cal system Σ does not in general enable one to predict with certainty the result of an experiment on Σ , and that in particular one can never predict with cer- tainty both the position and the momentum of Σ (Heisenberg's Uncertainty Principle). It further asserts that most pairs of observations are incompatible, and cannot be made on Σ simultaneously (Principle of Non-commutativity of Observations)."

Empirical nonclassical logic cntd.

2. Foulis & Randall "Operational statistics" (1972) DOI 10.1063/1.1665890 "The purpose of the series of papers here begun is to erect a new mathematical foundation for an operational theory of probability and statistics based upon a generalization of the conventional notion of a sample space. In subsequent papers, we shall formally establish on this foundation the notion of a "physical system" and an affiliated "theory of measurement." This latter generalized theory of measurement should prove to be particularly useful in the developing behavioral sciences and in addition shed some light on the difficulties that surround the measuring process in quantum mechanics. ... We are prepared, for instance, to regard test procedures on an assembly line, data gathering processes (such as opinion polling), pencil and paper operations (such as executing computational algorithms), and even procedures involving subjective approvals or disapprovals as bona fide physical operations."

Decay & reconstruction of empirical logics by the pasting of contexts

Context A context or maximal observable is a collection of observables that is complete and mutually exclusive. It has a hypergraph representation as smooth curve. a_1 a_2 a_3

Pasting A **pasting construction** is a collection of contexts with possible intertwining contexts. It has a respective hypergraph representation.

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Anecdotal example: Firefly in a box

Two intertwining contexts with three mutually exclusive observables per context; e.g., David W. Cohen (1989) DOI 10.1007/978-1-4613-8841-8, Dvurečenskij, Pulmannová, KS DOI 10.5169/seals-116747.



... as if that is not enough ...

🔹 interlude 💊

... in another (part of?) the talk enter probability theory ...

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 - Choice of the distribution depends on the physical / psychological *etc* realization of the *BOO*.

Anecdotal example: probabilities on a cyclic logic whose respective hypergraph is a pentagon aka pentagram aka house



1) classical probability distributions in terms of convex combinations of the 11 twovalued states thereon;

2) quantum probability distributions according to Born, Gleason, and Lovász;

3) exotic probability according to Gerelle & Greechie & Miller (1974) and Wright (1978)

So far we only spoke about comparing different probability distributions on fixed collections of (interwined)observables ...

🔊 interlude 💊

... now we shall be talking about "weird" nonclassical collections of (interwined)observables ...

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Inseparability 101: Kochen & Specker's demarcation criterion 1967, Theorem 0 of DOI: 10.1512/iumj.1968.17.17004

Theorem 0. Let \mathfrak{N} be a partial Boolean algebra. A necessary and sufficient condition that \mathfrak{N} is imbeddable in a Boolean algebra B is that for every pair of distinct elements a, b in \mathfrak{N} there is a homomorphism $h: \mathfrak{N} \to Z_2$ such that $h(a) \neq h(b)$.



Hypergraphs with nonseparable set of two-valued states third column is Kochen & Specker (1967, Γ_3)



KS, DOI:10.1103/PhysRevA.103.022204

Hypergraph with nonunital set of 6 value assignments



Fig. 2 'Almost' Greechie diagram of a suborthoposet of $L(\mathbb{R}^3)$ without a unital set of two-valued states [e.g., $12\overline{1} = \operatorname{Sp}(1, 2, -1)$].

Josef Tkadlec, DOI:10.1023/A:1026646229896 based on Erna Clavadetscher-Seeberger, Diss. ETH Zürich (Specker) handle ETH: 20.500.11850/138142 based on Schütte's letters to Specker, April 22nd, 1965 & November 3rd, 1983 (communicated to KS by Specker).

Hypergraph with exotic contextuality derived from coloring

Hypergraph of biconnected intertwined contexts representing complete graphs with a separating set of 6 two-valued states which is non-partitionable: G_{32} , cf. Figure 6, p. 121 Greechie (1971) DOI: 10.1016/0097-3165(71)90015-X



Mohammad H. Shekarriz & KS, vertex labeling by partitions of $\{1, 2, 3, 4, 5, 6\}$ with no faithful orthogonal representation arXiv:2105.08520.

Thank you for your attention!



